SPECIFIC CYCLOTRON CORRELATIONS UNDER SPACE CHARGE EFFECTS IN THE CASE OF A SPHERICAL BEAM

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Abstract

High intensity primary ion beams at GANIL are necessary to induce high radioactive production rates in the frame of the SPIRAL project. In this paper, we show that an intense beam can be tuned at injection in a cyclotron so as to result in a spatially spherical beam in the machine, with a reduced halo formation.

1 INTRODUCTION

The question on high intensity beams in cyclotrons is of great interest (Stammbach [1]).Various new applications require a fine beam tuning and a good comprehension of the space charge effects in order to limit the halo formation, and to avoid beam losses and activation in the machine. First, we establish the exact matched solution in the academic case where the electric space charge force is linear. Then we present a self-consistent approach allowing us to take into account the non-linear effects. Finally, we present simulation results obtained in the case of our compact injector C01.

2 LINEAR ANALYSIS

We consider a reference particle (q,m) rotating without acceleration on a circle according to the equations :

$$x_0(t) = r_0 \cos(\omega t)$$
$$y_0(t) = r_0 \sin(\omega t)$$

The mid plane is represented by (x,y), and the magnetic field is reduced to a negative component b_z , so that the central particle turns counter clock :

$$\omega = \frac{p_0}{mr_0} = -\frac{qb_z}{m} = -\frac{qb_z}{m_0\gamma}$$

Around the central particle, a bunch of particles creates a repulsive electric field, which is supposed linear. Being interested in what happens in (x,y), we admit that there exists vertical focusing forces compensating the vertical repulsion. Moreover, we consider the mass $m=m_0\gamma$ of each particle to be constant, although its energy will vary due to the space charge effects. The coupled equations can then be written :

$$m\ddot{x} = qk(x - x_0) + qb_z\dot{y}$$
$$m\ddot{y} = qk(y - y_0) - qb_z\dot{x}$$

This gives in the complex plane, using $z = x + iy$:

$$\ddot{z} - i\omega\dot{z} - \lambda z = -\lambda z_0 \quad ; \quad \lambda = qk/m$$
$$z = A\,e^{r_1 t} + B\,e^{r_2 t} + r_0\,e^{i\omega t}$$
$$r^2 - i\omega r - \lambda = 0 \quad ; \quad r = r_1, r_2$$

The solution is stable for r_1,r_2 purely imaginary, which leads to the following condition on the intensity :

$$0 \le u = \frac{I}{I_{\max}} = \frac{4\lambda}{\omega^2} < 1 \;;$$

$$I_{\max} = \frac{2\pi^2}{\mu_0 c^2}\frac{f_{hf}^2\,|b_z|}{h}\Delta r^3$$

We can use now the matrix form :

$$\begin{bmatrix}\Delta x\\ \Delta y\\ \Delta\dot{x}\\ \Delta\dot{y}\end{bmatrix}(t) = \begin{bmatrix}x - x_0\\ y - y_0\\ \dot{x} - \dot{x}_0\\ \dot{y} - \dot{y}_0\end{bmatrix}$$

$$= \begin{bmatrix}U & V\\ \lambda V & U - \omega JV\end{bmatrix}\begin{bmatrix}\Delta x_0\\ \Delta y_0\\ \Delta\dot{x}_0\\ \Delta\dot{y}_0\end{bmatrix} = T_L\begin{bmatrix}\Delta x_0\\ \Delta y_0\\ \Delta\dot{x}_0\\ \Delta\dot{y}_0\end{bmatrix}$$

$$U_{11} = (-\mu_2\cos(\mu_1 t) + \mu_1\cos(\mu_2 t))/(\mu_1 - \mu_2) = U_{22}$$
$$U_{12} = (\mu_2\sin(\mu_1 t) - \mu_1\sin(\mu_2 t))/(\mu_1 - \mu_2) = -U_{21}$$
$$V_{11} = (\sin(\mu_1 t) - \sin(\mu_2 t))/(\mu_1 - \mu_2) = V_{22}$$
$$V_{12} = (\cos(\mu_1 t) - \cos(\mu_2 t))/(\mu_1 - \mu_2) = -V_{21}$$
$$J_{11} = J_{22} = 0 \quad ; \quad J_{12} = -J_{21} = 1$$
$$\mu_1 = \omega(1 - \sqrt{1-u})/2 \quad ; \quad \mu_2 = \omega(1 + \sqrt{1-u})/2$$

The Lorentz variables are not conjugate, so that T_L is not a symplectic matrix. Introducing the vector potential A and the generalised impulsion P , we find :

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$$A_x = -b_z y/2 \quad ; \quad A_y = +b_z x/2$$
$$P_x = m\dot{x} + qA_x = m\dot{x} + m\omega y/2$$
$$P_y = m\dot{y} + qA_y = m\dot{y} - m\omega x/2$$

$$\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta P_x / p_0 \\ \Delta P_y / p_0 \end{bmatrix} = \begin{bmatrix} U - \omega VJ/2 & \omega r_0 V \\ -\frac{\omega(1-u)}{4r_0} V & U - \omega JV/2 \end{bmatrix} \begin{bmatrix} \Delta x_0 \\ \Delta y_0 \\ \Delta P_{x0} / p_0 \\ \Delta P_{y0} / p_0 \end{bmatrix}$$

The co-ordinates (x,P_x;y,P_y) being conjugate, the transfer matrix T is now symplectic and satisfies the relations :

$${}^tTJT = J \quad ; \quad {}^tT^{-1} = -JTJ \quad ; \quad J = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}$$

Although T is time dependent, we search an initial condition which *remains constant in time* :

$$\sigma_0 = T\sigma_0 {}^tT \quad \Rightarrow \quad T\sigma_0 = -\sigma_0 JTJ$$

The solution is a *diagonal* matrix σ_0 with :

$$\sigma_{11} = \sigma_{22} = \Delta x^2 = \Delta y^2 = \Delta r^2$$
$$\sigma_{33} = \sigma_{44} = (1-u)\left(\frac{\Delta r}{2r_0}\right)^2$$

Using now the co-ordinates (Δr,Δs,$\Delta p_r/p_0$,$\Delta p/p_0$, we obtain a beam matrix $\hat{\sigma}_0$ which is also constant in time :

$$E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad ; \quad \hat{T} = \begin{bmatrix} I & 0 \\ \frac{1}{2r_0}E & I \end{bmatrix}$$

$$\hat{\sigma}_0 = \hat{T}\sigma_0 {}^t\hat{T} = \begin{bmatrix} \Delta r^2 I & \frac{\Delta r^2}{2r_0}E \\ \frac{\Delta r^2}{2r_0}E & \Delta r^2 \frac{1-u/2}{2r_0^2} I \end{bmatrix}$$

This beam matrix represents the *stationnary "round beam"* : the disk radius is Δr=Δs, the radial divergence and the momentum spread must be equal , the specific correlations (Δs-$\Delta p_r/p_0$) and (Δr-$\Delta p/p_0$) must also be equal, all this depending on the intensity ratio given by the parameter u. The non correlated longitudinal and radial emittances are equal and vanish for u=1 :

$$\frac{\varepsilon_r}{\pi} = \frac{\varepsilon_s}{\pi} = \frac{\Delta r^2}{2r_0}\sqrt{1-u}$$

In the limit case u=1, the projections (Δs-$\Delta p_r/p_0$) and (Δr-$\Delta p/p_0$) become segments : the bunch is "laminar" and even behaves like a rigid body : for one position (Δr, Δs), there is only one possible ($\Delta p_r/p_0$,$\Delta p/p_0$), the bunch rotating around the central particle with the period $4\pi/\omega$.

The emittance equation can be written in another way :

$$\delta = \frac{2\pi^2}{\mu_0 c^2}\frac{f_{hf}^2 |b_z|}{h} \quad ; \quad u = \frac{I}{I_{max}} = \frac{I}{\delta \Delta r^3}$$
$$\Delta r^4 - \frac{I}{\delta}\Delta r - 4r_0^2(\frac{\varepsilon}{\pi})^2 = 0$$

For given emittances and intensity, the ideal "round beam" radius is deduced from this 4th order polynomial. This is similar to the Kapchinsky-Vladimirsky-Lapostolle result, except that $\Delta\phi$ is not considered to be constant, which explains the term in Δr instead of Δr^2 in the equation. If we consider an adiabatic acceleration of the round beam, the disk radius remains constant. This produces a linear decrease of the phase length.

Knowing that the only external force is the radial focusing, and that the space charge force is repulsive, it may be surprising that the longitudinal length does not increase. In fact, due to the initial conditions and the slow vortex motion, all the particles profit from the radial focusing "from time to time". For intermediate values of the parameter u, each particle trajectory is composed of a rotation around a centre rotating itself around the central particle, which gives a rosette-like figure.

3 SELF-CONSISTENT APPROACH

In order to study the non-linear space charge effects, we have built a program called LIONS_SP, merging our code LIONS [2], used for the study of the CIME cyclotron, with our code CHA3D [3], which is a 3D Poisson solver using the conjugate gradient algorithm.

3.1 Characteristics of the calculations

LIONS_SP uses the classical Particle In Cells method. It is optimised in Fortran 90 to work either on vectorial or parallel computers, using optionally the HPF extensions.

In order to obtain a good precision, the number of macro particles is chosen about 600000. The Poisson computer box, which follows the central particle, is a 30 mm cube, with a mesh containing about 3.4 millions points. With such parameters, we use about 1.2 Gigabytes of central memory, and the CPU time is typically 3 minutes per turn in the cyclotron, on a FUJITSU VPP5000.

3.2 Influence of the density distribution.

The previous analysis allows us to build an initial “round beam” for a given couple emittance-intensity. However, in the linear case, the charge density is supposed to be constant inside the bunch and zero outside. In the code, we distribute in a uniform way the N particles in the 6D phase space. In the 3D real space, this gives a sphere of radius Δr with a more realistic distribution :

$$\rho = \frac{8}{\pi^2}\frac{qN}{\Delta r^3}\left(1-\frac{\delta r^2}{\Delta r^2}\right)^{\frac{3}{2}} \qquad ; \qquad 0 \le \delta r \le \Delta r$$

The spherical Poisson equation can also be solved analytically, and with given radius Δr and intensity, it generates a slope $E(\delta r)/(\delta r)$ at $\delta r=0$ which is $32/3/\pi$ times greater than the corresponding slope in the linear case. Launching such an initial bunch with LIONS_SP, we observe an increasing disk radius, followed by its pulsation around the initial Δr. Moreover, if we chose N in order to obtain the linear slope, the disk radius begins to decrease, and pulses. In fact there is an intermediate value Δr which avoids the radius pulsation. With such an adjustment, and using all the conditions given by the linear analysis, the bunch remains a stationary round beam *without any halo formation*. If we accelerate the beam, its radius remains constant, and *the phase length decreases*.

3.3 Various initial conditions

We have studied the beam behaviour for various *unmatched* initial conditions :

- If the longitudinal length is chosen longer than Δr, it appears a spiral galaxy shape, with two tails.
- If the bunch is round, but with an energy spread smaller than in the linear theory, a spiral galaxy shape appears also, but with two tails oriented in the opposite direction.
- If we don’t apply the required (Δs-$\Delta p_r/p_0$) and (Δr-$\Delta p/p_0$) correlations, the mixing of a disk deformation and a halo formation appears.

All these tests prove that the linear analysis provides a good initial beam matching which minimises the halo.

Moreover, the matching conditions without any space charge (u=0) give an excellent way to tune a round beam, with a phase length decreasing in the machine.

4 APPLICATION TO CYCLOTRON C01

The cyclotron C01 is one of the two injectors used at GANIL to accelerate stable beams in cascade with CSS1 and CSS2. In the frame of the THI project [4], it is presently “revisited”. For this purpose, we have recalculated the magnetic field using TOSCA, and the RF 3D field using CHA3D.

The C01 has 2 characteristics which complicate the tuning : there are 3 spiral shape poles, which gives no field symmetry. It is a compact cyclotron with an axial injection, so that it is not easy to find the ideal injection matching in backward, due to the transit time in the first gaps. However, to tune the beam in the presence of space charge effects, we proceed the following way : first, we construct an initial condition one turn before the extraction, using the “round beam” linear conditions and the best (Δr-$\Delta p_r/p_0$) and (Δz-$\Delta p_z/p_0$) correlations inherent to the machine. Then we go backward using the “linear option” of the code, down to the injection point, where we filter the particles which are inside the theoretical round beam momentum spread. We deduce all the correlations of this sub-bunch and start forward with a new set of particles. Then, using the PIC method, we can check the “round beam” behaviour in the machine. The whole process provides the relevant injection correlations which must be created by the low energy transfer line, in order to create the round beam in the C01.

There is however one drawback making difficult a real “round beam experiment” in the C01 : the required momentum spread at injection is greater than what can give our buncher. A rebuncher installed about 1 meter before the injection could be the solution to obtain all together *the desired phase length* and *the mometum spread*, and increase significantly the intensity. Although the actual intensity obtained in the C01 is sufficient to achieve the 6 Kilowatts required in the THI project, its increase could allow us to cut in emittance between C01 and CSS1, and make easier the tuning in CSS2.

5 CONCLUSION

The linear “round beam” approach allows us to find out the exact matching which must be satisfied as an initial condition, for given emittances and intensity. The radial and longitudinal emittances must be equal, and the correlations (Δs-$\Delta p_r/p_0$) and (Δr-$\Delta p/p_0$) must be set according to the ratio I/I_{max}. Moreover, the non-linear PIC method confirms the linear analysis.

The power of the actual computers allows us to solve the Poisson equation with great precision, provided that the algorithm is implemented in a parallel language.

Finally, the linear “round beam” approach provides a good way to design a high intensity cyclotron, or a cascade of cyclotrons, with their associated beam lines and bunchers.

It gives also an interesting way to tune a cyclotron even in the absence of space charge effects.

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