BEAM OPTICS AND DYNAMICS OF FFAG ACCELERATORS
- Application of TPSA and Polymorphism -

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Abstract

As a versatile accelerator, especially for application with high repetition and high current beams, the FFAG (Fixed Field Alternating Gradient) synchrotron attracts growing attention. We will discuss the design procedure and beam dynamics study from a modern view based on recent progress in high-energy accelerators. Comparison between brute force tracking with direct use of 3D field mapping data and a newly introduced way modeling the entire field region with a collection of thin elements is presented.

1 INTRODUCTION

Optics design and dynamics study of FFAG (Fixed Field Alternating Gradient) accelerators involve, finding the closed orbit, calculation of linear lattice functions, estimate of parameter dependence, and search of dynamic aperture. Unlike a synchrotron and other accelerators, large nonlinearities are inherent in FFAG in order to make the transverse tune constant in a large momentum range. Higher order optical effects such as amplitude dependent tune shift have to be estimated in the early design stage. Since the first development of FFAG in the MURA days, there has been little serious activity. We will discuss the application of modern tools to FFAG study, for example, the construction of a transfer map for analysis and perhaps tracking studies.

2 DESIGN OF FFAG OPTICS

2.1 Some Basics of FFAG Optics

In a beam-focusing channel using the alternating gradient principle (or strong focusing principle), the focusing properties such as phase advance per unit cell depend on particle momentum. That is called the chromaticity. It cannot be corrected if the system is a purely straight channel. However, in a circular channel with bending magnets, the closed orbit, meaning the orbit without betatron oscillations, depends on particle momentum. The derivative of the closed orbit with momentum is called the dispersion function. Using the dispersion function and nonlinear magnetic elements, we can make the chromaticity zero. For example, if we install a sextupole magnet where the dispersion function is finite, the closed orbit of a higher momentum particle is shifted

outwards, and linear focusing strength expanded at the sextupole with respect to the closed orbit is added. In the same way, linear focusing strength due to the sextupole for a lower momentum particle is subtracted.

The FFAG principle, more strictly speaking its scaled version, assures the compensation of chromaticity over a large momentum range just by shaping the bending magnets to obtain median plane fields as follows

$$B(r,\theta) = B_i \left(\frac{r}{r_i} \right)^n F(\vartheta),$$

where r is the radius from the machine center, B_i and r_i are the bending field and the radius corresponding to injection momentum. The F depends on a type of FFAG. The so-called radial sector type satisfies

$$F(\vartheta) = 1,$$

and spiral sector type does

$$F(\vartheta) = F\left(\theta - h \ln \frac{r}{r_i} \right),$$

where θ is azimuthal angle. With using those fields, two conditions that give the zero chromaticity can be described as

$$\left. \frac{\partial}{\partial p} \left(\frac{K}{K_0} \right) \right|_{\vartheta = cos\, t.} = 0$$

- Geometrical similarity.
- Constancy of n at corresponding orbit points.

$$\left. \frac{\partial n}{\partial p} \right|_{\vartheta = const.} = 0$$

where p is momentum K is the local curvature, K_0 is average curvature, and ϑ is generalized azimuth.

In alternating gradient magnets, bending fields with opposite signs are positioned alternatively, which makes the closed orbit scallop shape as shown in Fig. 1. Since the normal bending magnet (it bends orbit inward) gives horizontal focusing, it is called a focusing (F) bend. The reverse bending magnet is called a defocusing (D) bend. Notice that the closed orbit does not go through at a constant B field, so that the local curvature is varying along the orbit.

A radial sector type shown in Fig. 1 is in its original configuration. As a variation, a radial sector with triplet focusing structure is also considered [2]. That can be

obtained by splitting the D bend (smaller magnet) into two and making them adhere to the adjacent F bends. As a result, longer straight sections between two D bends are obtained. Also field-cramping effects are expected between F and D bends.
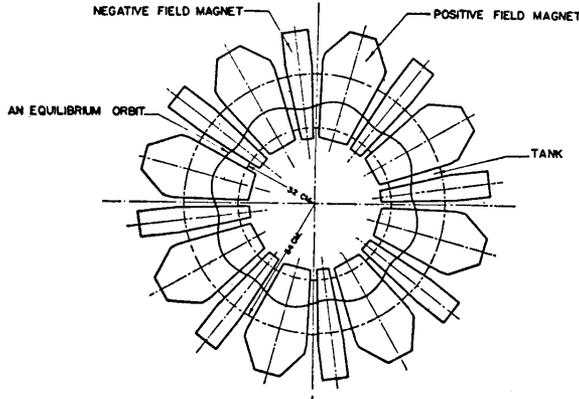

Fig 1: Cited from Ref. 1 (first electron FFAG model). Larger magnets have normal bending fields and bend a particle inward; smaller ones have reverse bending fields and bend a particle outward. As a result, the central orbit has a scallop shape.

2.2 Design Procedure

Now we describe the optics design in a FFAG accelerators. The procedure certainly resembles that of an ordinary synchrotron and a cyclotron, but some aspects deserve special emphasis. We limit ourselves to a scaled FFAG, namely one that satisfies the zero chromaticity condition. The necessary items should be the following:
- Find the closed orbit.
- Calculation of linear lattice function.
- Estimate parameter dependence.
- Search for the dynamic aperture.

In a FFAG synchrotron, the ideal closed orbit is not just a combination of straight lines and arc of circles as we have seen above, while it is usually the case in an ordinary synchrotron. A sector focus cyclotron is somewhat similar to the FFAG, but the variation of local curvature in a magnet is much smaller since the field index n cannot be as large as FFAG to keep the isochronous condition. Ultimately, numerical integration is necessary to obtain the closed orbit.

Although a closed orbit cannot be quickly computed, focusing properties such as phase advance and betatron amplitude functions are estimated with a linearized model of the rather complicated magnet. In the model, we assume constant curvature in F and D bends, respectively. At the same time, we assume that the linear gradient along the orbit is constant, so that both bending magnets becomes just like a combined function magnet with a bending field and a constant quadrupole field. One thing we cannot ignore even in this simplified model is the edge

focusing which can be the primary focusing strength in the vertical direction. The linearized orbit with some geometrical quantities is depicted in Fig. 2. A standard synchrotron lattice design code such as SAD [3] is then employed to obtain a rough idea of focusing parameters [2].
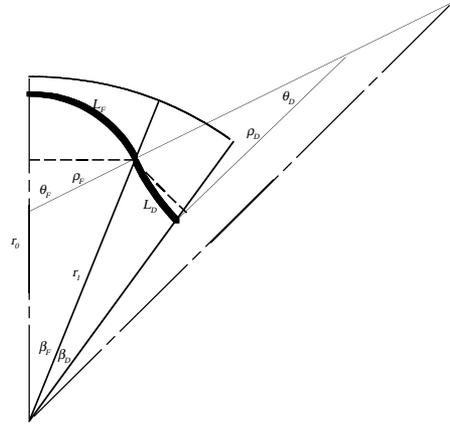

Fig 2: One half sector of the triplet radial sector FFAG. In order to estimate the closed orbit and focusing properties in a quick manner, the magnetic field is simplified (linearized) so that the arc has a constant curvature in F and D bends, respectively. Linear focusing strength is also the same along the orbit.

Nonlinearities in FFAG are not just perturbations, but the main components to force zero chromaticity for a large momentum range, say a factor of three. That is quite different from an ordinary synchrotron where zero chromaticity is only assured for a few percent deviation of momentum at most. Such strong nonlinearities make the transverse tune a function of betatron amplitude even at small amplitude. The condition of zero chromaticity is also sensitive to the realistic fringe fields and that cannot be taken into account in the linearized model. Based on the linearized model, rough parameters such as the field index and the azimuthal angle of each magnet are determined. However, a 3D calculation of the magnetic field and numerical integration using them are necessary at some point to accurately model the machine. That is an iterative process. Nowadays, it is straightforward to calculate the 3D field mapping using a code like TOSCA. We will discuss numerical integration in more detail.

The final step is to search for the dynamic aperture just to make sure that it is large enough. In the horizontal direction, the physical aperture limit does not exist in practice. We can stack beams filling up a dynamic aperture. Therefore an enlargement of dynamic aperture directly ends up with higher particle intensity.

2.3 Numerical Integration

For each item, with the exception of the calculation of the linearized model, step-by-step integration is eventually necessary to make an accurate design for construction.

The most primitive way is a direct numerical integration using interpolated magnetic fields based on the 3D grid data by TOSCA. For example, in order to find the closed orbit, first we guess phase space coordinate where the closed orbit most probably exists. Then, we launch a particle, integrate it step by step, and see if the particle comes back to the same position after one turn or one minimum period. Usually the first guess does not match the closed orbit. Then what we do is to track a particle for more turns. Unless the tune is a rational number, trajectory in a phase space becomes ellipse. The center of the ellipse should be more probable position of the closed orbit. An iterative process is necessary.
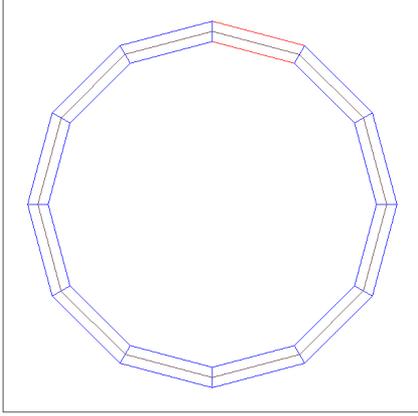

Fig. 3: Floor plan of the 12 sectors of FFAG in PTC. Each identical sector (a trapezoid) is divided to many thin elements based on the calculation of 3D magnetic fields.

The similar step by step integration and iteration can be more systematically performed with PTC (Polymorphic Tracking Code) package recently developed by Forest and Schmidt [4]. Instead of interpolating the 3D field data from adjacent grids every time step, we first replace the entire 3D field mapping of one period with a bunch of thin elements aligned in longitudinal direction. In each thin element, three magnetic field components are fitted either globally or locally so that its derivatives with respect to the coordinates exist to the necessary order. Once the 3D fields are represented in that manner, real or Taylor type (since it is determined at run time and therefore it is called polymorphic) can be integrated.

Let us take the same example, namely finding a closed orbit. Now suppose the first guess X of the closed orbit is different from the goal by ε. By definition of a closed orbit, $X+\varepsilon$ does not change by the transformation T, where T is succession of a thin element operation,

$$T(X+\varepsilon)=X+\varepsilon,$$

By taking the first order only,

$$T(X)+\frac{\partial T}{\partial X}\varepsilon=X+\varepsilon,$$

then ε becomes

$$\varepsilon=\left(\frac{\partial T}{\partial X}-1\right)\!\left(X-T(X)\right)\cdot$$

Of course, iteration is still necessary until ε becomes small.

In fact, the primary virtue of taking a polymorphic code is its ability to make normal form. Once the 3D magnetic field mapping is replaced with a bunch of thin elements and a transformation of phase space coordinate including its derivatives through those elements is obtained, then we can construct normal form that is given by

$$R=A^{-1}\circ M\circ A,$$

where M is the one-turn (one-period) map, A is the normalizing transformation, and R is a rotation.

Using that transformation, the β-function is nothing but

$$\beta=A_{11}^2+A_{12}^2\cdot$$

Parameter dependence such as tune as a function of transverse coordinates as well as momentum spread and of their higher orders are obtained automatically. In PTC, parameters are not just phase space coordinates, but other machine parameters such as the strength of magnetic fields.

2.4 Trajectory in Phase Space

We will compare the tracking results of step by step integrations and Taylor map tracking. As an example, 150 MeV FFAG under construction [5] is modeled. The kinetic energy is fixed at 27.4 MeV. The operating point is (3.80, 1.19). Since the 3rd order structure resonance is at $\nu_x=4$, strong resonance structure in phase space appears in Fig. 4. Dynamic aperture is roughly estimated as the maximum closed curve. That is the result of step by step integration. Figure 5 shows the horizontal tune depending on amplitude.
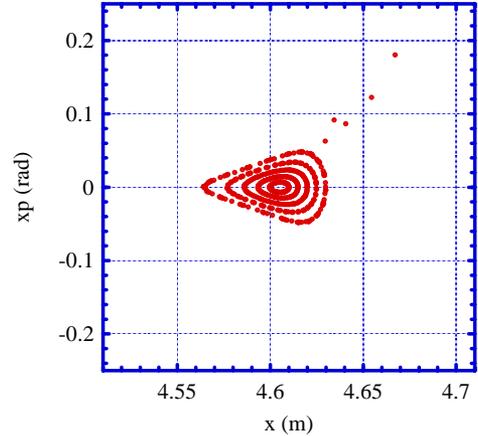

Figure 4: Step by step integration result. The 4th order Runge-Kutta method is employed and horizontal phase space is depicted.
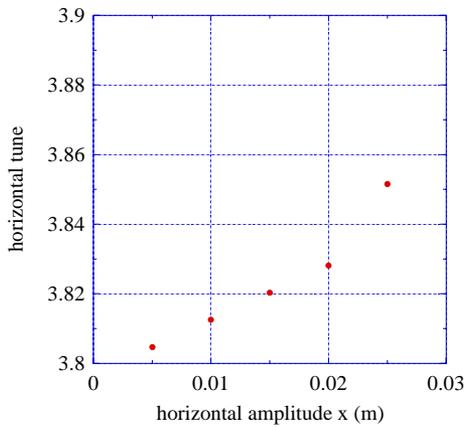
Fig. 5: Amplitude dependence of the horizontal tune. Five points correspond to surviving particles in Fig. 4.

Now, Fig. 6 shows tracking result of PTC with the same machine parameters. The scale is about the same as Fig. 4. There are two different kinds of trace. One whose color is red is the result directly from the map after successive integration of thin elements. The yellow color corresponds to a generating function map. The significant width in the red trace disappears in the yellow one. The phase space behavior and dynamic aperture calculated by step-by-step integration turns out not so different from the Taylor map in this particular example.
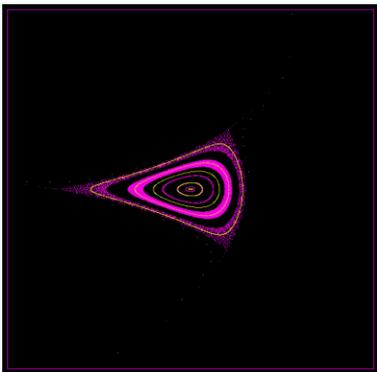

Fig. 6: Tracking results from the map based on successive integration of thin elements. The scale is about the same as Fig. 4.

3 SUMMARY

Using a recently developed the PTC package by Forest, a FFAG synchrotron was modeled and the results were compared with a brute force step-by-step integration. In fact, it turns out that brute force tracking is not so inaccurate in that particular example although the tracking takes more time and the book keeping is more cumbersome. We plan to extend the FFAG modeling with PTC to handle acceleration where the closed orbit shifts as momentum increases.

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