

## THE PROPAGATION OF ANNULAR IREBS IN PERIODIC PERMANENT MAGNETIC (PPM) FIELD

Yang Jian-hua, Zhang Ya-zhou, Zhang Jian-de, Liu Jin-liang, Li Chun-lu  
National University of Defense Technology, Changsha, China, 410073

### Abstract

The feasibility of propagating annular IREB in periodic permanent magnetic (PPM) field was studied. The magnetic field profile of alternatively arranged permanent magnet rings was calculated first using the finite element magnetic method (FEMM). The forces acting on IREB's electron in such magnetic field were analyzed by the use of fluid model and the radial force equation in modified Mathieu function form was drawn then. At last a 2.5-D particle in-cell (PIC) simulation code was used to investigate the physical process of the IREB's propagation. The results showed that the transverse and axial momentum of beam electrons were modulated and the amplitude of beam envelope fluctuation was relative to the beam intensity and the beam electron's initial incident angle. The condition of the minimum beam envelope scallop was obtained and conclusion was made that several kilo-amperes intense annular electron beam could propagate stably in a guiding periodic magnet with commercial permanent magnets.

**Keywords:** Annular electron beam, finite element magnetic method (FEMM), periodic permanent magnet (PPM), PIC simulation

High-power microwave (HPM) as a new technology has potential applications in diverse fields such as directed-energy weapon, plasma heating, radio-frequency electron linac powering, laser pumping, etc, the power levels of HPM sources have reached to 15GW at the wavelength of 3 cm. Nevertheless such HPM sources, being general massive and expensive devices, are not suitable to be put into practical uses. One of the problems with them was that they have to use the strong magnetic field of pulsed magnetic coils or superconducting magnets. So, it is very important to study the possibility of guiding the IREB's propagation by permanent magnetic field. Apart from compactness and cheapness, permanent magnetic field devices would have the merit of repetitive operation. It was reported that coaxial PPM had been used as wiggler and focusing system for ubitrons to produce microwave in Stanford Linear Accelerator Center (SLAC), and microwave power of 250MW was generated by 500kV, 1kA electron beam.

In section 1, the PPM was analyzed with finite element magnet method. The equilibrium equation and the stability condition of an annular electron beam

propagation in PPM is obtained using a fluid model in section 2. The particle simulation method is used to investigate the physical process of the transportation of electron beam guided by PPM and some results are shown in section 3.

### 1. The Calculation of PPM

The finite element magnet method (FEMM) was used to study the PPM. For low-frequency problems addressed by FEMM, the displacement currents could be ignored.

FEMM was composed of 3 parts:

1. Preprocessor. This is a CAD-like program for laying out the geometry of the problem to be solved and defining material properties and boundary conditions.
2. Solver. The solver would take a set of data files that describing the problem and solve the relevant Maxwell's equations to obtain values for the magnetic field through the solution domain.
3. Postprocessor. This is a graphical program that displayed the resulting fields in the form of contour and density plots. The code also allows the user to inspect

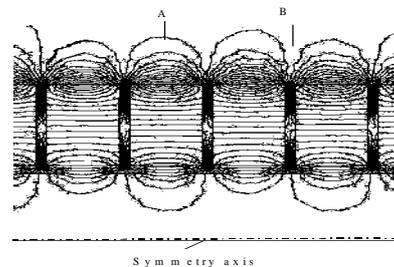


Fig.1 the profile of magnetic field

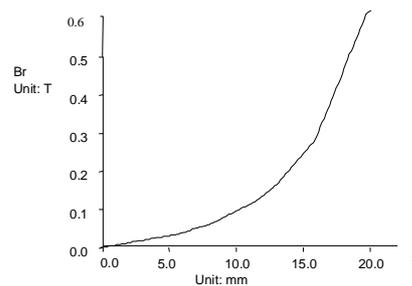


Fig.2 radial magnetic flux density versus the radius at section B

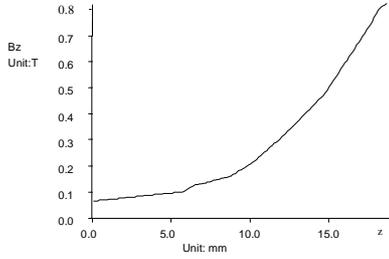


Fig.3 Axial magnetic flux density versus the radius at section A

the field at arbitrary points and plot various quantities of interest along user-defined contours.

The problem was defined as follows: there were 13 periods with 0.2 cm length of soft magnet and 3cm length of hard magnet in one period and the inner radius of the PPM was 2.5cm. The hard permanent material was NdFeB with magnetic product 37MGOe and was considered as a linear material with permeability 1.048, the soft magnet was nonlinear B-H with the permeability of 14600. The divided cells in hard and soft magnetic material were smaller than the cells in the air.

The main calculation results were shown in Figs.1-3. Comparing Fig.2 and Fig.3 with  $I_0(k_\omega r)$  and  $I_1(k_\omega r)$ , it was found that the first harmonic field of PPM was dominant. The field of PPM could be approximately expressed as  $B_r(r) = B_\omega I_1(k_\omega r)$ ,  $B_z(r) = B_\omega I_0(k_\omega r)$ , where  $B_\omega \approx 0.07T$ .

## 2. The Theoretical Analysis of Propagation of Electron Beam Through PPM

According to the charged fluid model, there would be four kinds of forces acting on electron beam element.

The space-charge force was

$$F_{rs} = \frac{e^2 n_0}{\epsilon_0 \gamma^2} (r - r_0) = \frac{m \omega_p^2}{\gamma^2} (r - r_0) \quad (1)$$

Where  $\omega_p$  was the plasma frequency. The centrifugal force was

$$F_{rc} = \frac{e^2 B_r^2(r)}{\gamma m k_\omega^2 r} \cos^2(k_\omega z) + \frac{P_0^2}{\gamma m r^2} / r + \frac{2e B_r(r) P_0}{\gamma m r k_\omega} \cos(k_\omega z) \quad (2)$$

Where  $P_0$  was the initial azimuthal momentum. The focusing lorentz force was

$$F_{rm} = -\frac{e^2 B_r(r) B_z(r)}{\gamma m k_\omega} \cos^2(k_\omega z) \quad (3)$$

At the zero-order beam equilibrium, when  $P_0=0$  there would be

$$\frac{e^2 B_r^2(r)}{\gamma m k_\omega^2 r} \cos^2(k_\omega z) - \frac{e^2 B_r(r) B_z(r)}{\gamma m k_\omega} \cos^2(k_\omega z) - \frac{m \omega_p^2}{\gamma^2} (r - r_0) = 0 \quad (4)$$

By the approximation  $B_r(r) = B_\omega I_1(k_\omega r)$ ,  $B_z(r) = B_\omega I_0(k_\omega r)$ , (4) reduces to

$$\gamma m \frac{\Omega_\omega^2}{\gamma^2 k_\omega} \cos^2(k_\omega z) f(r) - \frac{m \omega_p^2}{\gamma^2} (r - r_0) = 0 \quad (5)$$

where  $f(r) = -I_1^2(k_\omega r)/k_\omega r + I_1(k_\omega r) I_0(k_\omega r)$ ,  $\Omega_\omega = \frac{e B_\omega}{m}$

The equilibrium beam radius  $r_{ce}$  could be determined from(4). When the beam element was apart from its equilibrium radius  $r_{ce}$  by a small interval,  $f(r)$  could be expressed as

$$f(r) = f(r_{ce}) + g(r_{ce})(r - r_{ce}),$$

where  $g(r_{ce})$  was a  $r_{ce}$  related constant and the radial force equation became

$$\frac{d^2 r}{dt^2} + \frac{\Omega_\omega^2}{2\gamma^2 k_\omega} [1 + \cos(2k_\omega z)] g(r - r_{ce}) - \frac{\omega_p^2}{\gamma^3} (r - r_{ce}) = 0 \quad (6)$$

(6) could be rewritten as

$$\frac{d^2 \sigma}{dZ^2} + \alpha [1 + \cos(2Z)] - \beta = 0 \quad (7)$$

Where  $Z = k_\omega z$ ,  $\sigma = 2(r - r_{ce}) / \Delta r_b$ ,

$$\Delta r_b = r_2 - r_1, \alpha = \frac{\Omega_\omega^2 g}{2\gamma^2 (k_\omega v_z)^2 k_\omega}, \beta = \frac{\omega_p^2}{\gamma^3 (k_\omega v_z)^2}.$$

Eq(7) was a modified Mathieu function. It showed that when  $\alpha = \beta$ , the scallop of beam envelope would take its minimum. That was to say the optimum stability condition for the annular relativistic electron beam through the PPM was

$$\frac{\omega_p^2}{\gamma} \frac{k_\omega}{g(r_{ce})} = \frac{\Omega_\omega^2}{2} \quad (8)$$

which was relative not only to the beam plasma density and its equilibrium radius but also to the PPM intensity and its period length.

### 3. The Particle-in-cell (PIC) Simulation of Beam Transportation Through PPM

The PIC simulation method was an effective, intuitional and significant approach to investigate the movement of the electrons in the drift tube. In the present simulation, a 2.5 dimension model with cylindrical coordinates was introduced. A  $-500\text{kV}$  voltage wave transmitted to diode, and the electrons

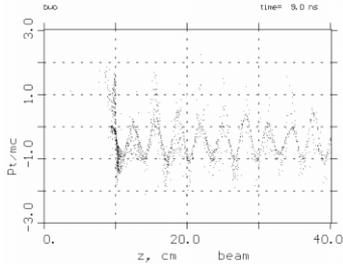


Fig.4 The radial velocity vs. z

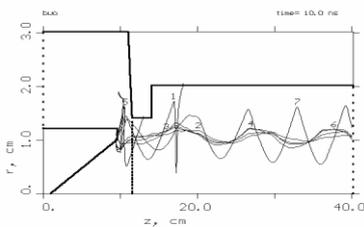


Fig.6 The scheme of some typical macro-particles of electron

emitted from the cathode by self-consistent particle emission model. The value of the magnetostatic field in diode and drift tube region at each cell was evaluated by a data file, which was produced by FEMM.

Some results were given by the simulation:

- 1.The axial and radial velocity of electrons would be modulated when they traveling in PPM (Fig.4 and Fig.5)
- 2.Annular electron beam would have better laminar flow than solid beam, an annular beam would carry more current than that of a solid beam in the same conditions. As an example, in the case of 1cm beam radius 2kA current could be transported by annular beam and only 1.7kA current by solid beam.
- 3.In the same magnetic field, the propagated current would increase with the increasing of the cathode radius. This could be explained that at large radius the increasement of focusing force was quicker than that of the repulsive forces.
- 4.It was found that those electrons would lose in the wall soon, when they were incident from diode to drift tube with an initial angle of bigger than  $20^\circ$ . The electrons would keep better laminar flow and less minimum scallop when the incident angles were smaller. The picture shown in Fig.7 was a good notation. The incident angle of the macro-particle of electron 1 and 5

larger than  $20^\circ$ , so they couldn't propagate far away. Particle 7 had bigger incident angle, this led to bigger amplitude oscillation and smaller oscillation period. So there were some electrons lost during the beam propagation. Particles 2, 3, 4, 6 could keep nearly the same oscillation amplitude because their incident angles were smaller. The beam in the tube was shown in Fig.7.

5. The beam guided by PPM was used in backward wave oscillator (BWO) to produce microwave in PIC

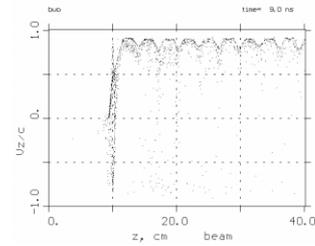


Fig.5 The axial velocity vs. z

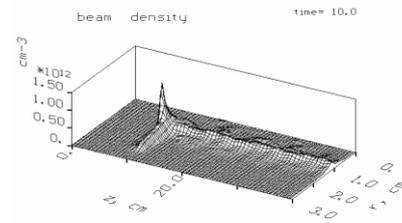


Fig.7 Three dimension geometry of electron beam

simulation test. More than 100MW of TM mode at X band microwave was produced, which showed the PPM approach in beam guiding was a possible way in HPM to decrease the weight and volume of the system.

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