Vertical Orbit-Excursion FFAGs (VFFAGs) and 3D Cyclotrons

I. Principle & Magnetic Fields

II. Proton Driver Study

III. Isochronous 3D Cyclotrons

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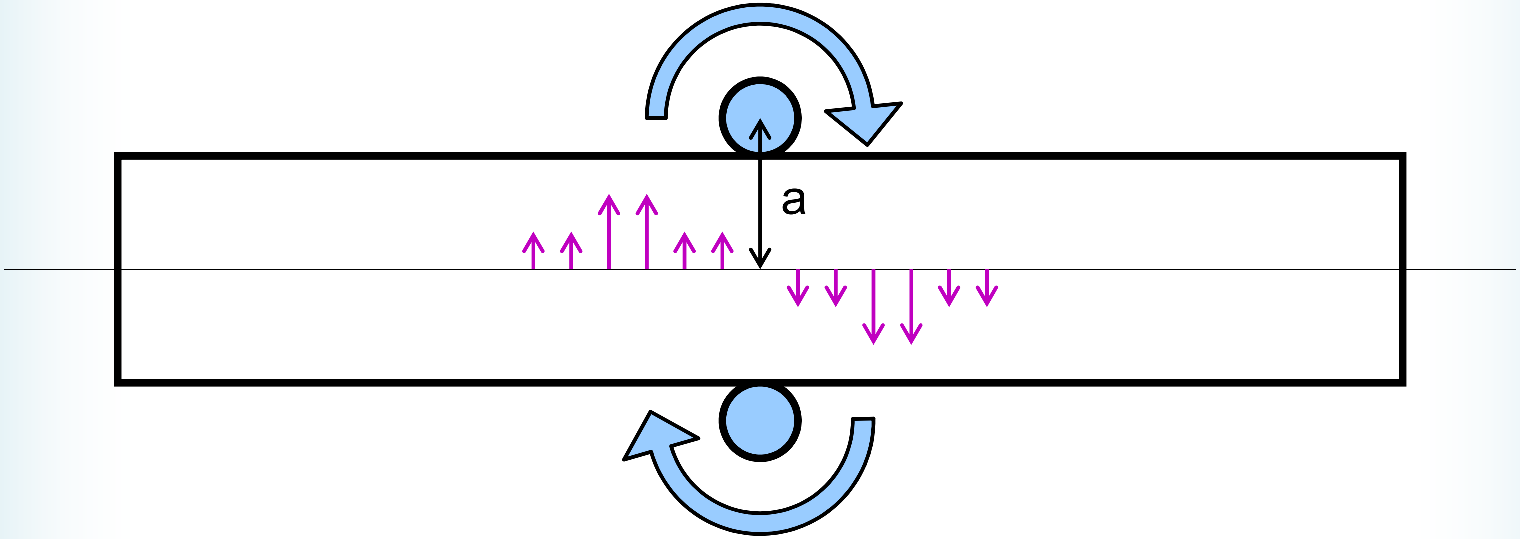
I. Principle & Magnetic Fields

In an FFAG (or cyclotron) the orbit moves across the magnet aperture during acceleration

Horizontal Aperture SC Magnet

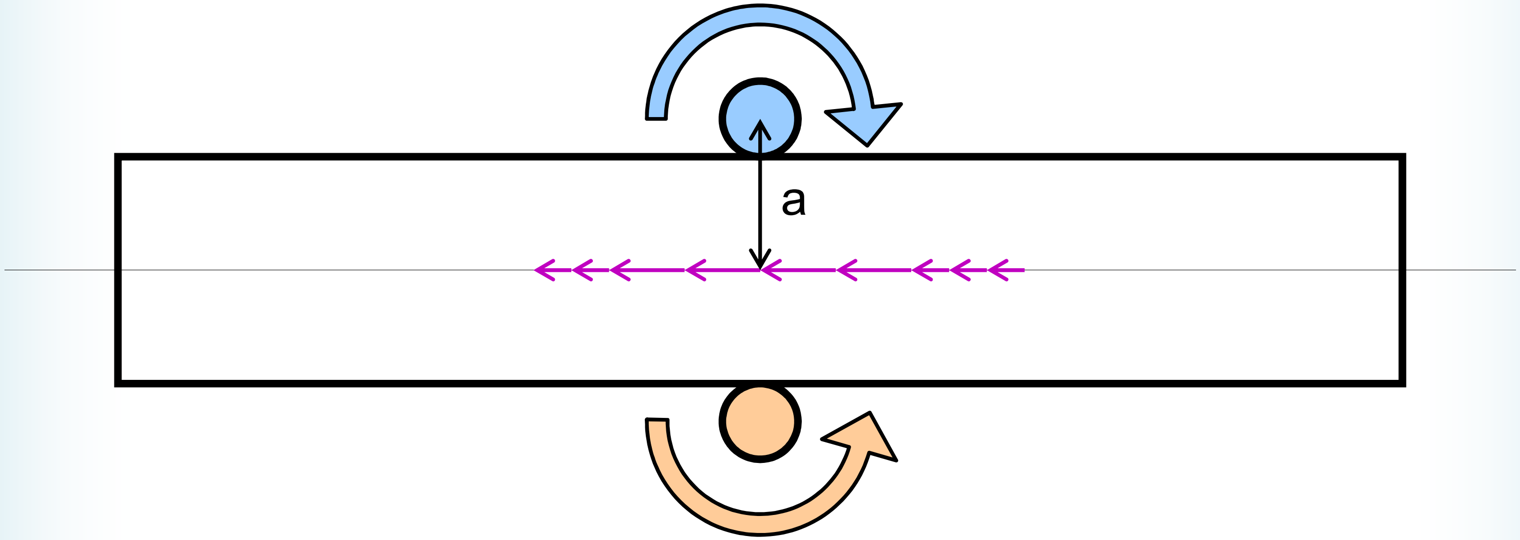
- Getting vertical **B** field requires same-direction current windings on opposing sides



- B_y proportional to $x/(a^2+x^2)$: cancels at $x=0$!

Constructive Interference

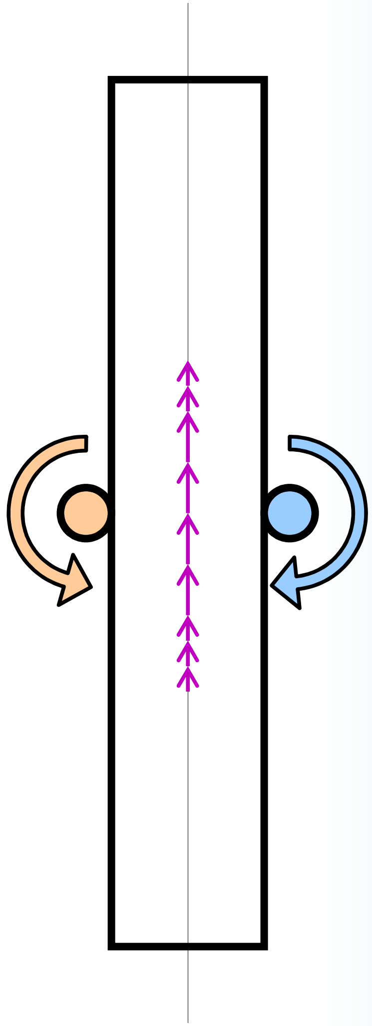
- Getting horizontal **B** field requires opposite current windings and is easier



- B_x proportional to $a/(a^2+x^2)$

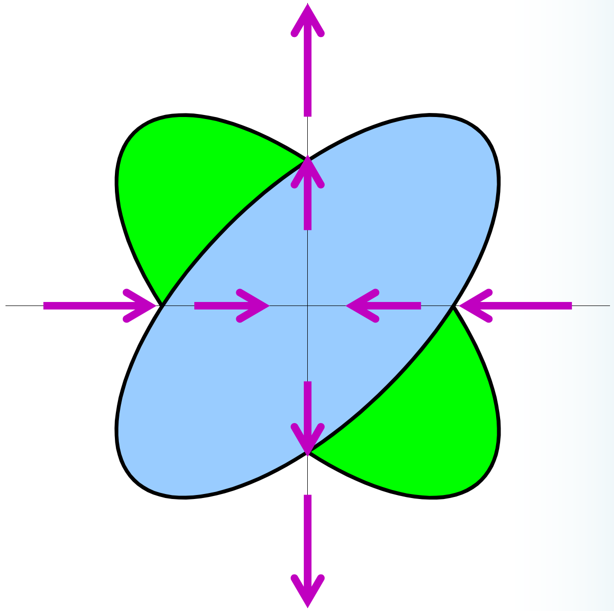
Vertical Aperture SC Magnet

- But now the field is in the wrong direction!
- That's OK, rotate the magnet
- The dipole field is there
- But what sort of focussing does this magnet give?



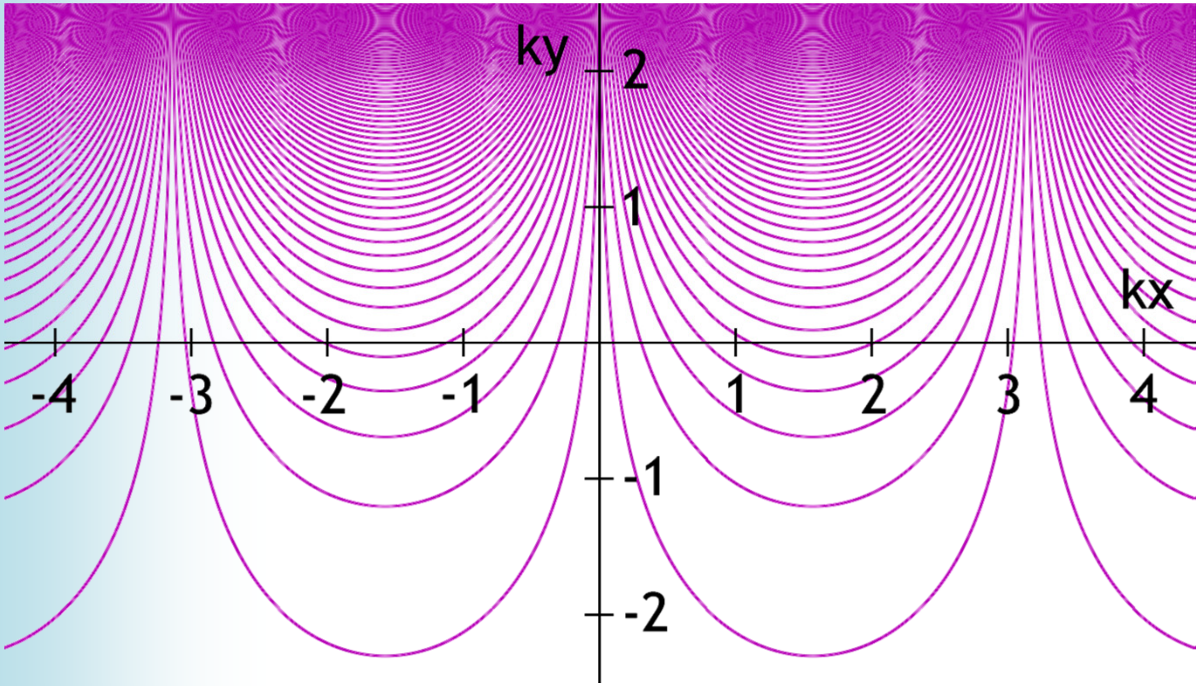
“Scaling” VFFAG Magnet

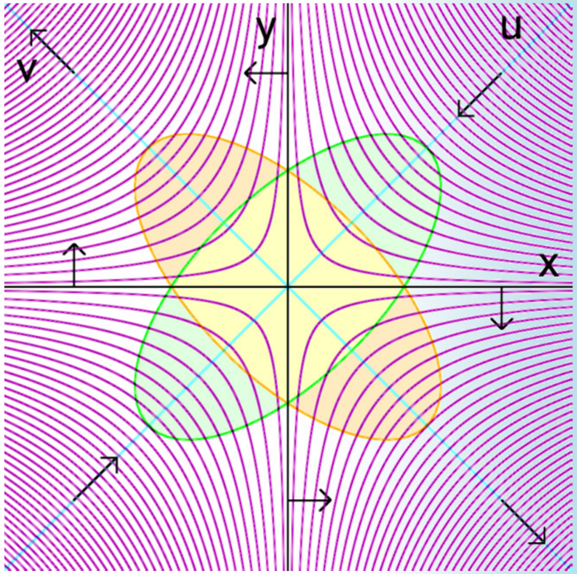
- Dipole field should increase moving up the magnet, so set $B_y = B_0 e^{ky}$ on axis (x=0)
- Subtracting dipole component leaves the field of a skew quad:
  - Exponential is good because moving upwards just scales the field and all gradients
  - Thus closed orbits at different momenta are exactly the same shape, just translated upwards
  - VFFAG = Vertical orbit excursion FFAG



Scaling VFFAG Field & Scaling Law

$$B_y = B_0 e^{ky} \cos kx \qquad B_x = -B_0 e^{ky} \sin kx$$





$$y \mapsto y + \Delta y, \qquad (p, \mathbf{B}) \mapsto (p, \mathbf{B})\mathrm{e}^{k\Delta y}$$

FODO Scaling VFFAG Machine

- First VFFAG tracking simulation, for HB2010
  - 2D, zero space charge, nonlinear magnets
- 150mm.mrad ε_{geom} input beam
- Proton-driver-like but nasty circumference factor! (C=17)

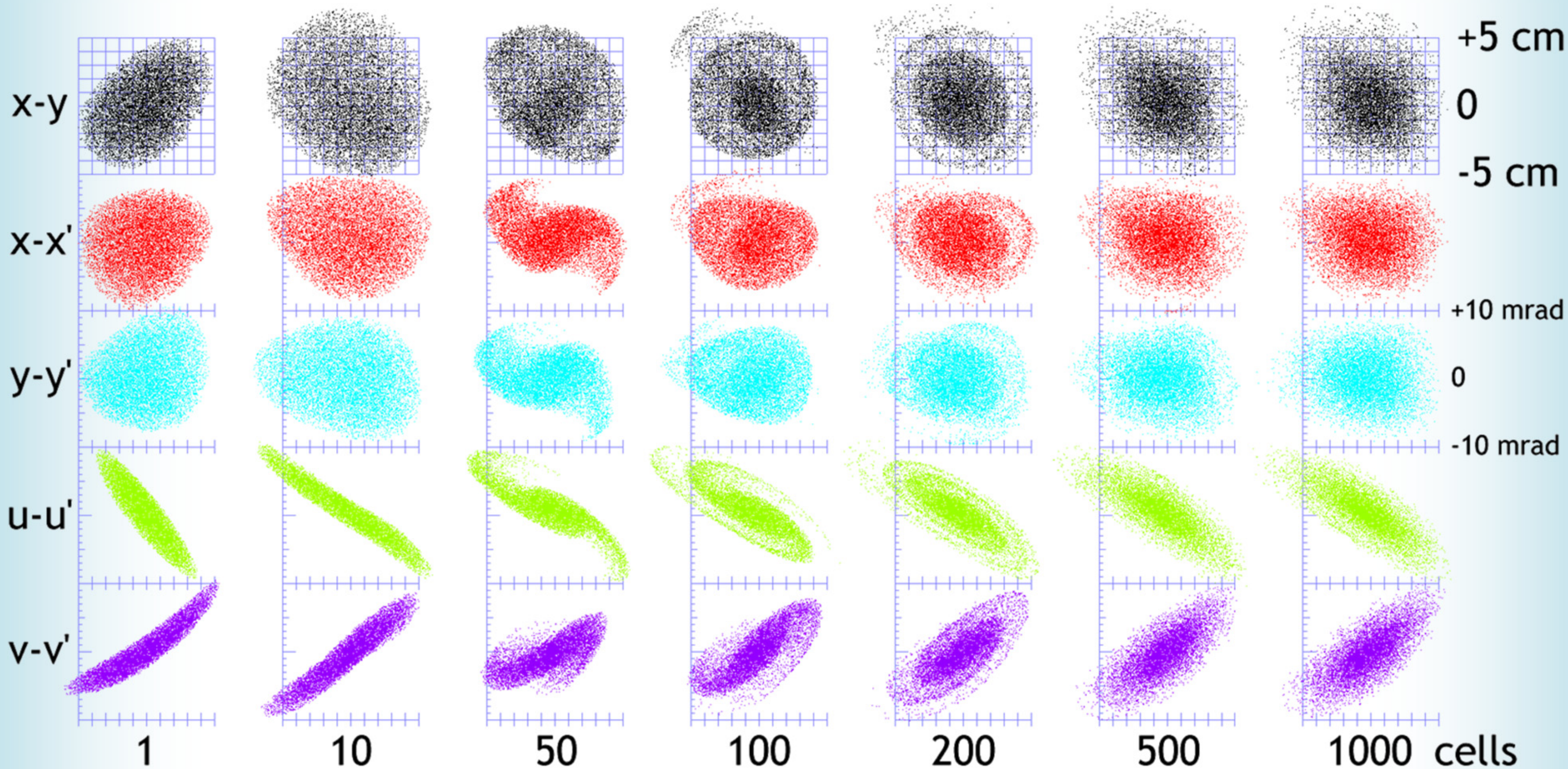
$$\boxed{C = \langle |\boldsymbol{B}| \rangle / \langle B_y \rangle}$$

Table 1: Parameters of the FODO lattice.

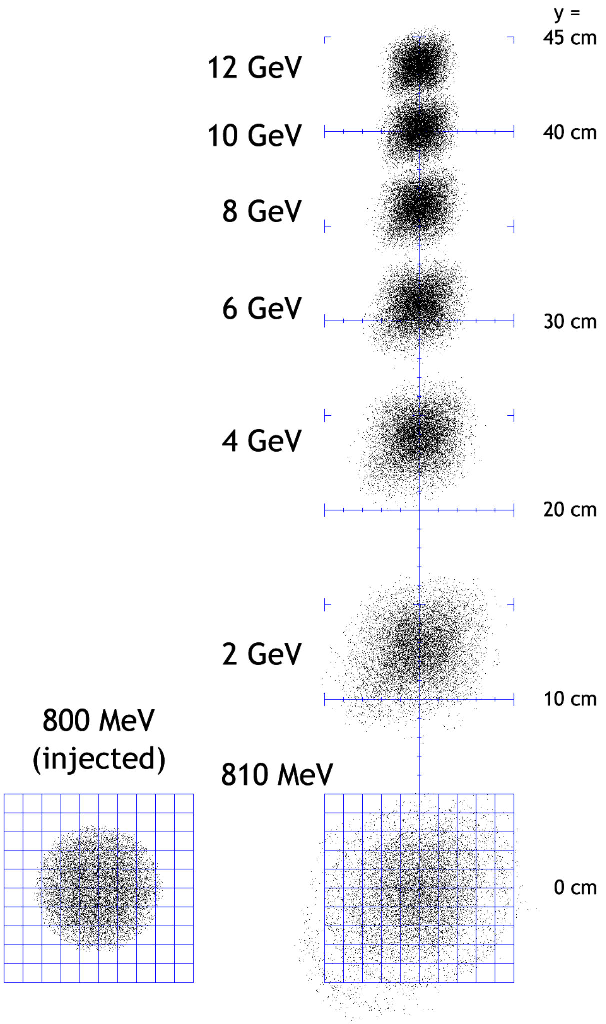
Energy range	800 MeV–12 GeV
Orbit excursion	43.5 cm (vertical)
k	$5\,\mathrm{m}^{-1}$
B_0	0.5 T
$B_{\max}$	4.41 T (beam centre)
	4.96 T (beam top)
	5.33 T (whole magnet)
Lattice	FODO
F length	0.4 m
D length	0.45 m
Drift length	4 m


MeV
tance=-1.798m
am=100%

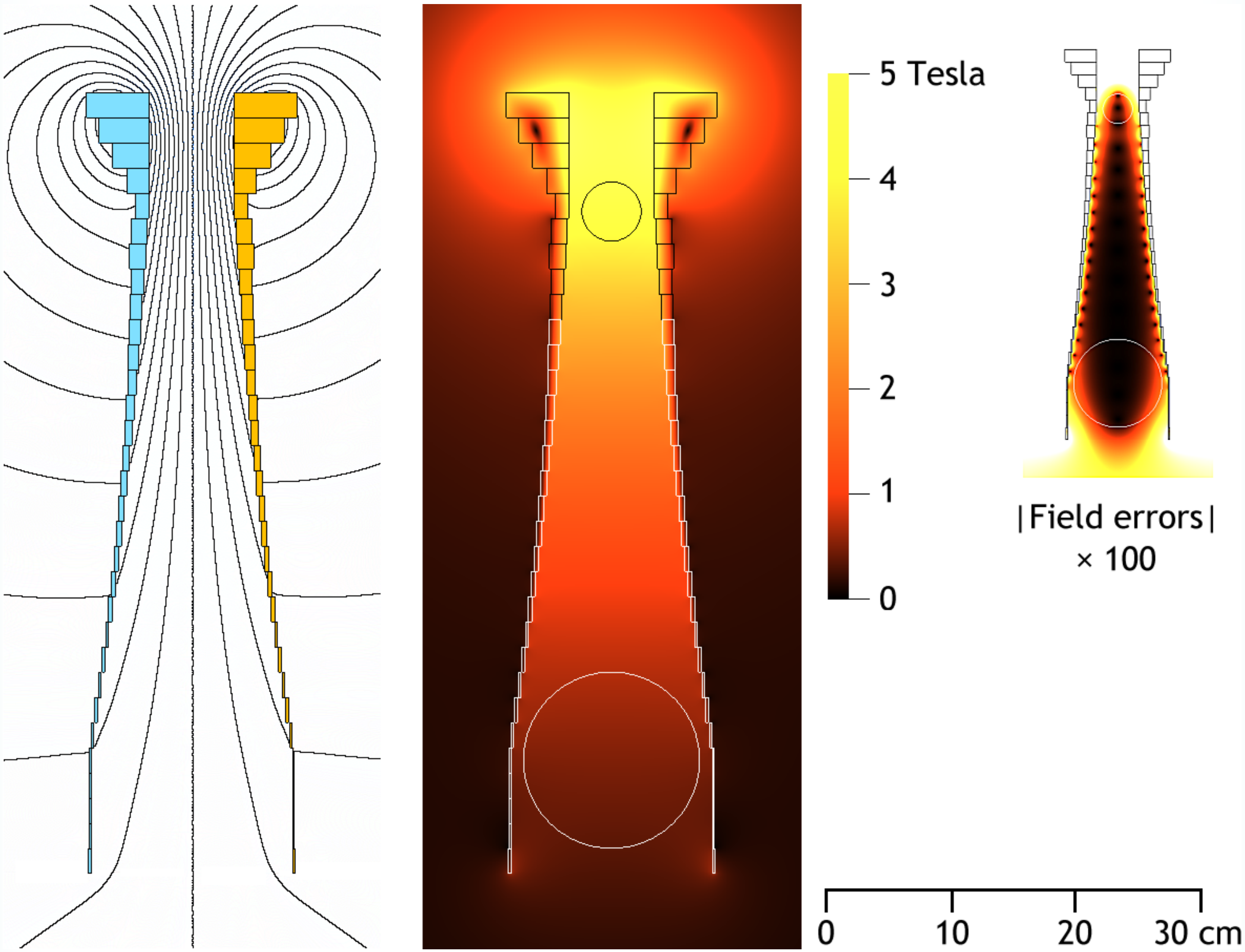
Scaling VFFAG with Mismatch



VFFAG Acceleration



2D Winding Model for Magnet



Application: Hadron Therapy?

- Low intensity but high rep-rate machines
  - Fixed field useful, space charge not too bad
- Small beams
  - The VFFAG magnet can be a narrow vertical slot
  - Less stored energy, smaller windings required
- Fixed tune allows slower acceleration, less RF
- Disadvantage: we still have the FFAG extraction-from-an-orbit-that-moves problem

Historical References

- "FFAG Electron Cyclotron" (Ohkawa, 1955)
  - T. Ohkawa, Physical Review **100** p.1247, abstract (1955)
  - Talk on isochronous electron VFFAG with exponential field, with and without edge focussing
- "Helicoidal FFAGs" (Leleux, 1959)
  - G. Leleux, J. Proy and M. Salvat, Rapport OC 70, Service de Physique Appliquee Section d'Optique Corpusculaire (1959)
  - Linear optics analysis of VFFAG
- "Accelerators with Vertically Increasing Field" (Teichmann, 1960-2)
  - J. Teichmann, translated from Atomnaya Énergiya, Vol.12, No.6, pp.475–482 (1962)
  - Isochronous, fixed tune electron VFFAG, exponential field, suggestion of curved orbit excursion

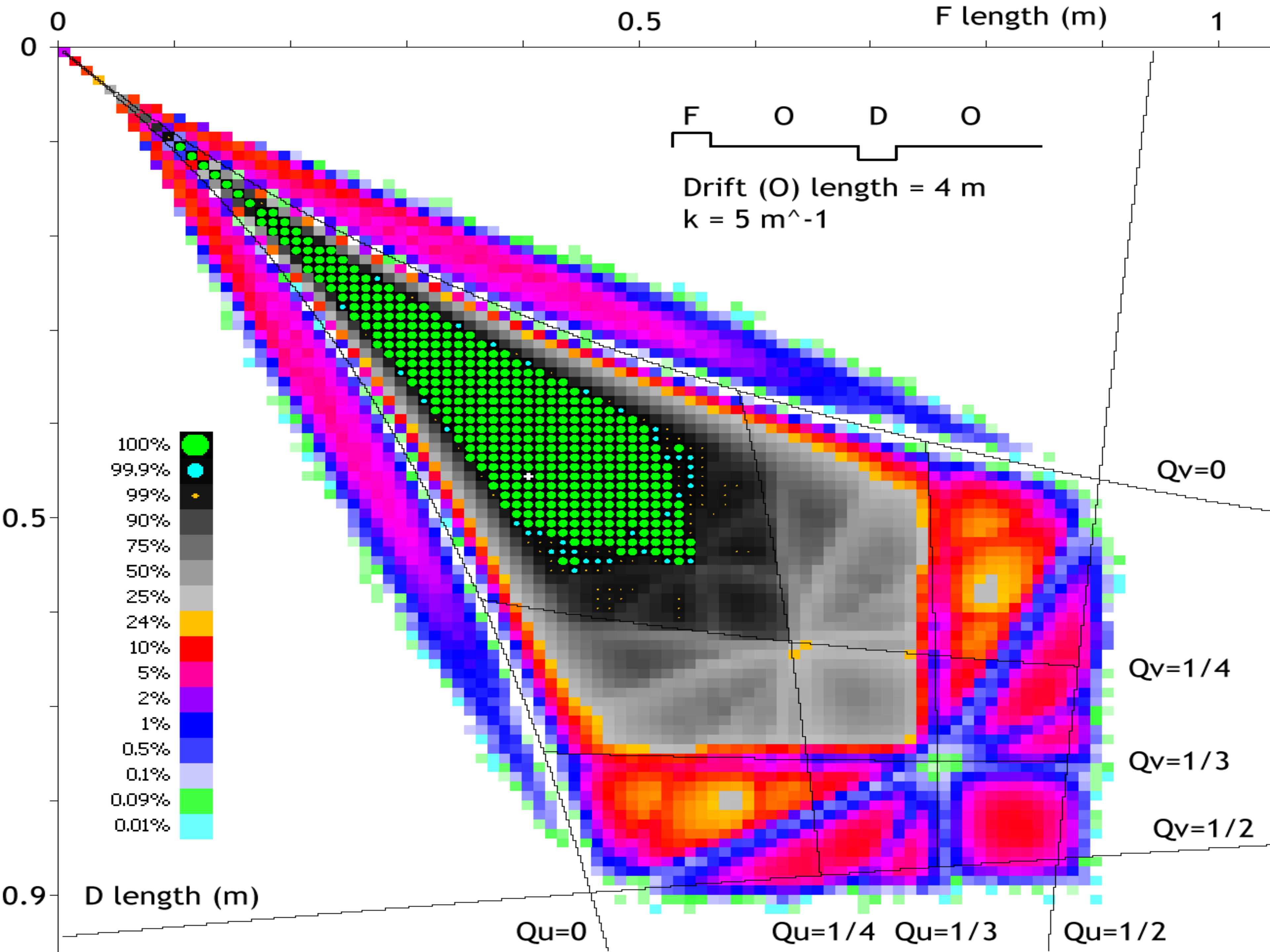
II. Proton Driver Study

Motivation: ISIS Energy Booster

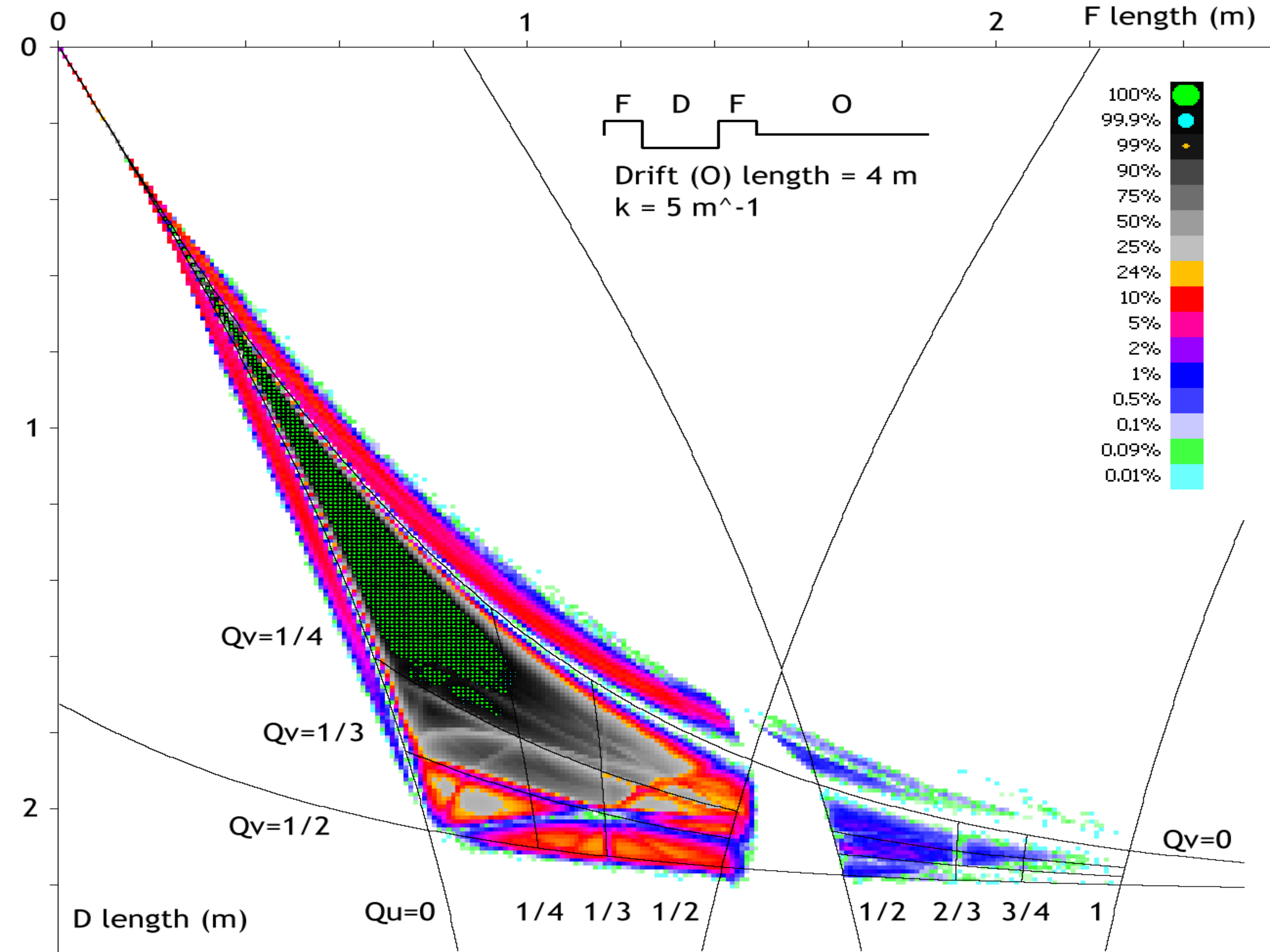
- FFAG to accelerate the 50Hz pulsed ISIS beam
- Energy: 800MeV – 12GeV
- Superconducting magnets
- Ring radius 52m (2x ISIS) could do 2.5x,3x
- Mean dipole field in magnets 0.47 – 4.14T
- 30% RF packing, 20% magnets, 2-4m drifts
- Warm 6.2 – 7.3MHz RF
- Harmonic number 8 (10,12 in larger ring)

Scaling (V)FFAG disease

- Defocussing requires reverse bending, as in scaling FFAGs → large circumference
- Searched for "lopsided" scaling VFFAG lattices with good dynamic aperture [HB2010]
  - 10000 particles were tracked for 1km
  - Survival rate plotted on axes of lengths of "F" and "D" type magnets
  - This reveals both the lattice stability region and resonance stop-bands



F length (m)
0
0.5
1
D length (m)
0.5
0.9
F O D O
Drift (O) length = 4 m
k = 5 m^-1
100%
99.9%
99%
90%
75%
50%
25%
24%
10%
5%
2%
1%
0.5%
0.1%
0.09%
0.01%
Qv=0
Qv=1/4
Qv=1/3
Qv=1/2
Qu=0
Qu=1/4
Qu=1/3
Qu=1/2

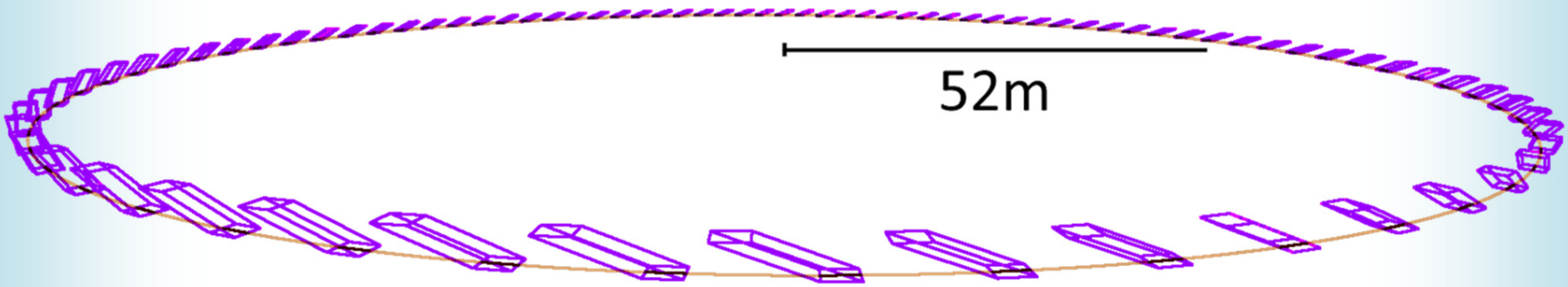


F length (m)
0
1
2
0
1
2
F D F O
Drift (O) length = 4 m
k = 5 m^-1
100%
99.9%
99%
90%
75%
50%
25%
24%
10%
5%
2%
1%
0.5%
0.1%
0.09%
0.01%
Qv=1/4
Qv=1/3
Qv=1/2
Qv=0
Qu=0
1/4
1/3
1/2
1/2
2/3
3/4
1
D length (m)

Lattices can't be very lopsided

- Unfortunately in all cases the region of dynamic stability sticks very close to the F=D diagonal line
- The 2nd FDF stability region does not have enough dynamic aperture
- So basic scaling VFFAGs will always be big, with much reverse bending
  - Could edge focussing avoid reverse bends?

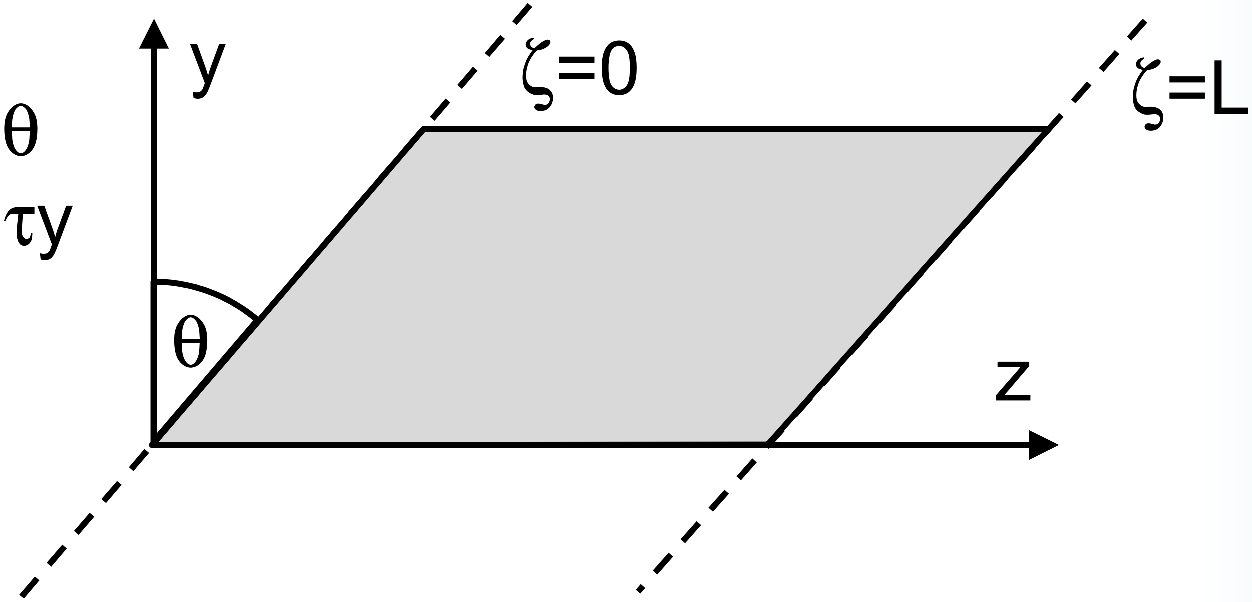
Vertical Edge Focussing [HB2012]



Superconducting magnets allow this to be smaller than synchrotron designs at lower energies

VFFAG with Edge Focussing

$\tau = \tan\theta$
$\zeta = z - \tau y$



one wants a mid-plane field $B_y = B_0 e^{ky} f(\zeta)$ but to obey Maxwell's equation $(\nabla \times \mathbf{B})_x = 0$, this has to be modified to $(B_y, B_z) = B_0 e^{ky} \left(f(\zeta) - \frac{\tau}{k} f'(\zeta), \frac{1}{k} f'(\zeta) \right)$.

Scaling law: $y \mapsto y + \Delta y, \qquad (p, \mathbf{B}) \mapsto (p, \mathbf{B}) e^{k\Delta y}$

$\boxed{z \mapsto z + \tau \Delta y}$

Spiral Scaling VFFAG Magnet Field

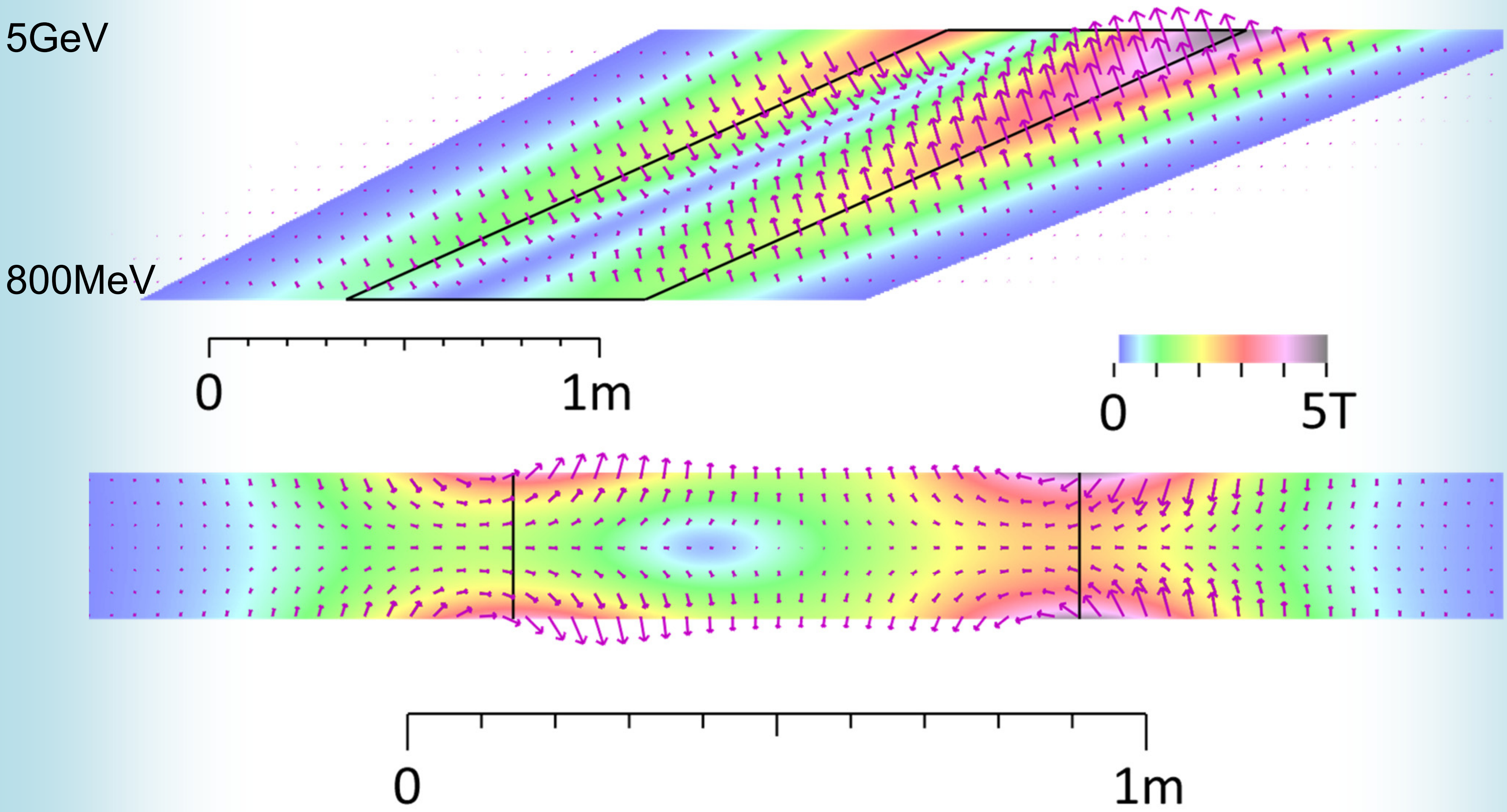


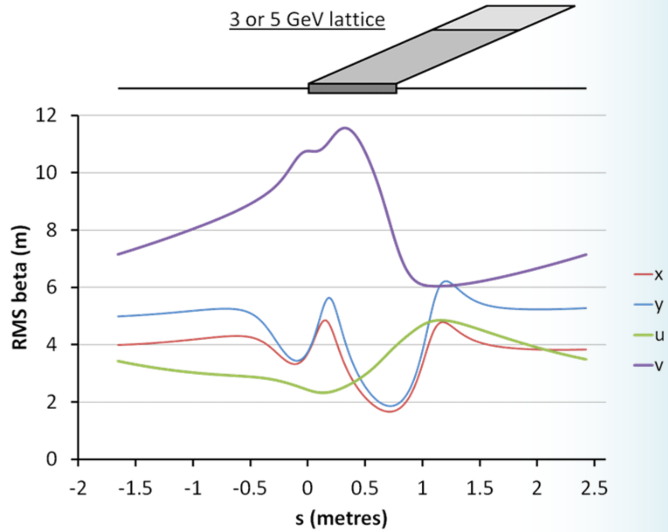
TABLE I. Transverse Parameters for VFFAG Rings

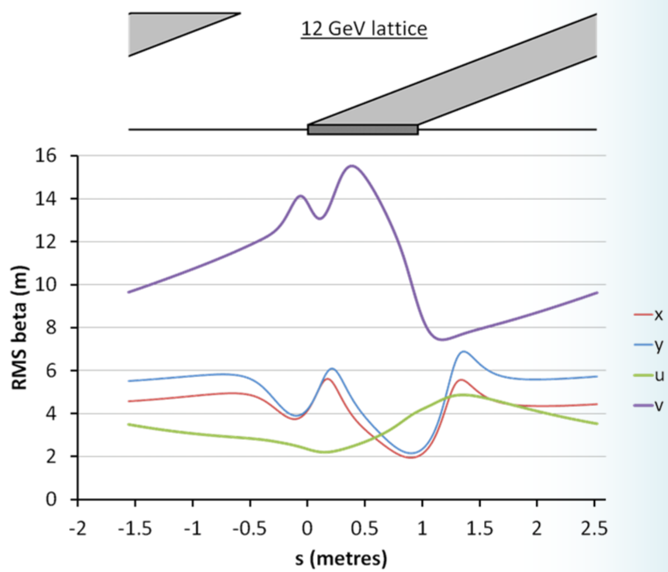
$E_{k,\mathrm{inj}}$	800 MeV		
$E_{k,\mathrm{ext}}$	3 GeV	5 GeV	12 GeV
Mean radius	52 m (2×ISIS)		
Superperiods	80 (superperiod is one cell)		
Cell length	4.0841 m		
Drift length	3.3174 m		3.1257 m
Magnet Parameters			
Magnet length	0.7667 m		0.9584 m
B_0	0.5 T		0.4 T
k	$2.01\,\mathrm{m}^{-1}$		$2.2\,\mathrm{m}^{-1}$
$\tau = \tan\theta_{\mathrm{edge}}$	2.23		2.535
θ_{edge}	65.84°		68.47°
Fringe length	$f = 0.3\,\mathrm{m}$ in $B \propto \frac{1}{2} + \frac{1}{2}\tanh(z/f)$		
B_{ext}	1.3069 T	2.0036 T	3.5274 T
$B_{\mathrm{fringe}}/B_{\mathrm{body}}$	$2.6941_{x=4\,\mathrm{cm}}$		$2.6174_{x=2\,\mathrm{cm}}$
$B_{\max}$	3.5210 T	5.3979 T	9.2326 T
Beam Optics			
$y_{\mathrm{ext}} - y_{\mathrm{inj}}$	0.4780 m	0.6906 m	0.9895 m
μ_u (per cell)	71.30°		71.29°
μ_v	28.65°		19.56°
Q_u (ring)	15.843		15.843
Q_v	6.367		4.347

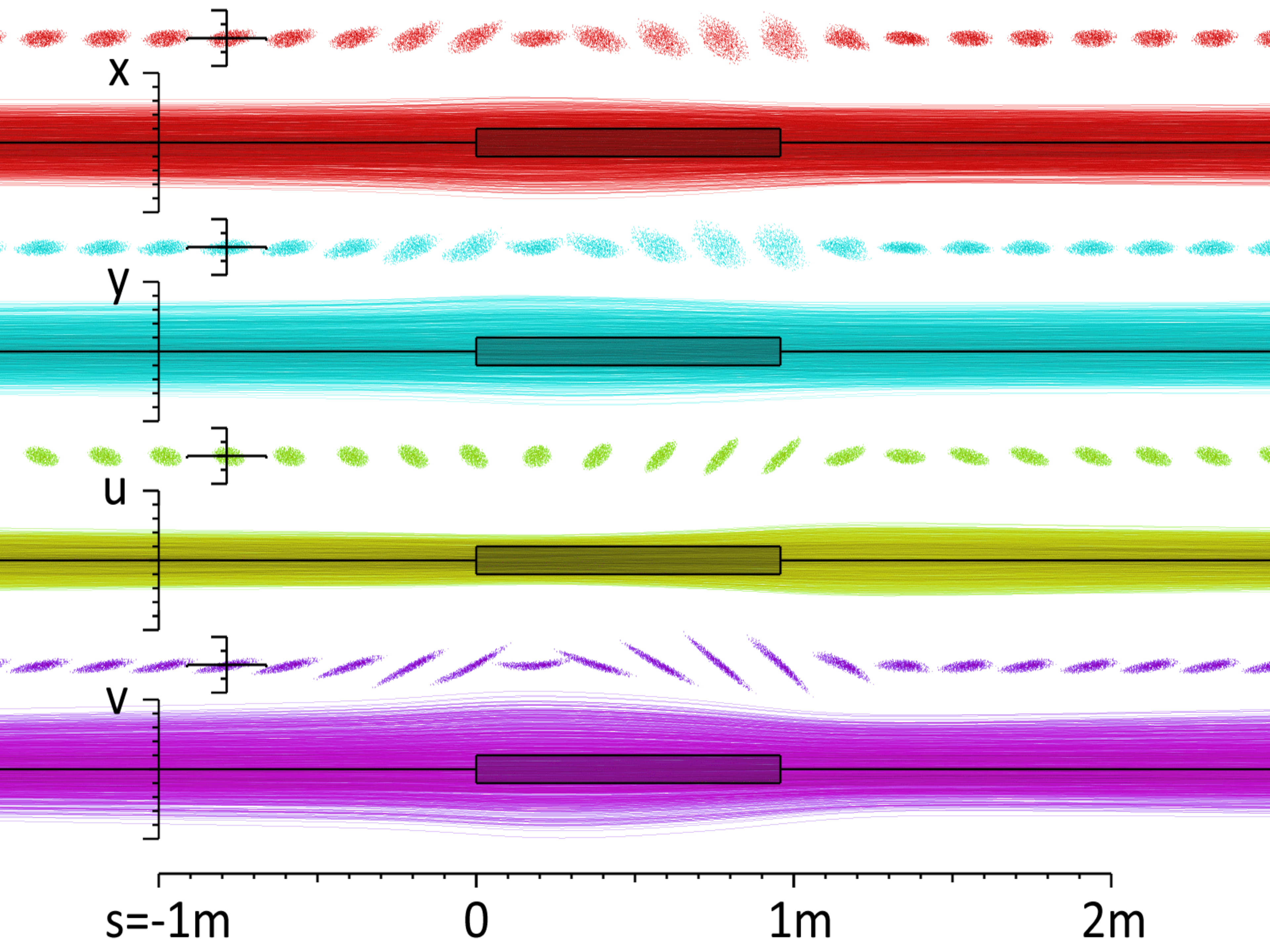
Cell Beta Functions

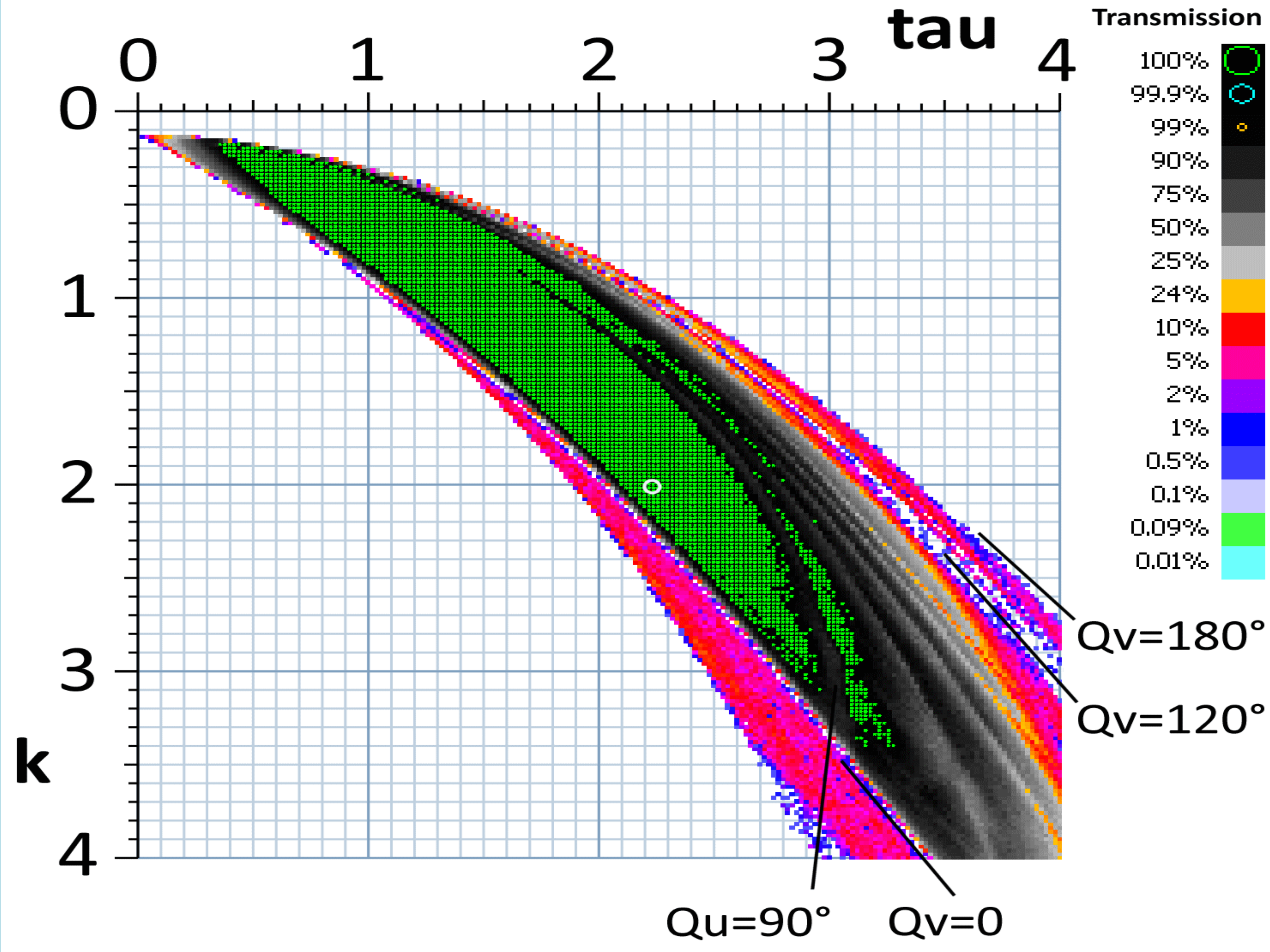
- Doublet focussing nature
  - Visible in u,v planes
- FfD
  - Doublet controlled by τ
  - Singlet controlled by k
- Ring tune sensitivity:

$$\frac{\partial Q_{u,v}}{\partial k} = \begin{bmatrix} -8.49 \\ -94.46 \end{bmatrix} \quad \text{and} \quad \frac{\partial Q_{u,v}}{\partial \tau} = \begin{bmatrix} 39.92 \\ 119.82 \end{bmatrix}$$



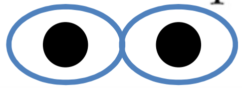


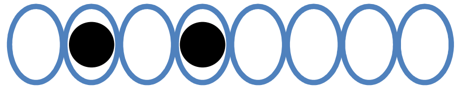

x
y
u
v
s=-1m
0
1m
2m



tau
Transmission
100%
99.9%
99%
90%
75%
50%
25%
24%
10%
5%
2%
1%
0.5%
0.1%
0.09%
0.01%
0
1
2
3
4
0
1
2
3
4
k
Qv=180°
Qv=120°
Qu=90°
Qv=0

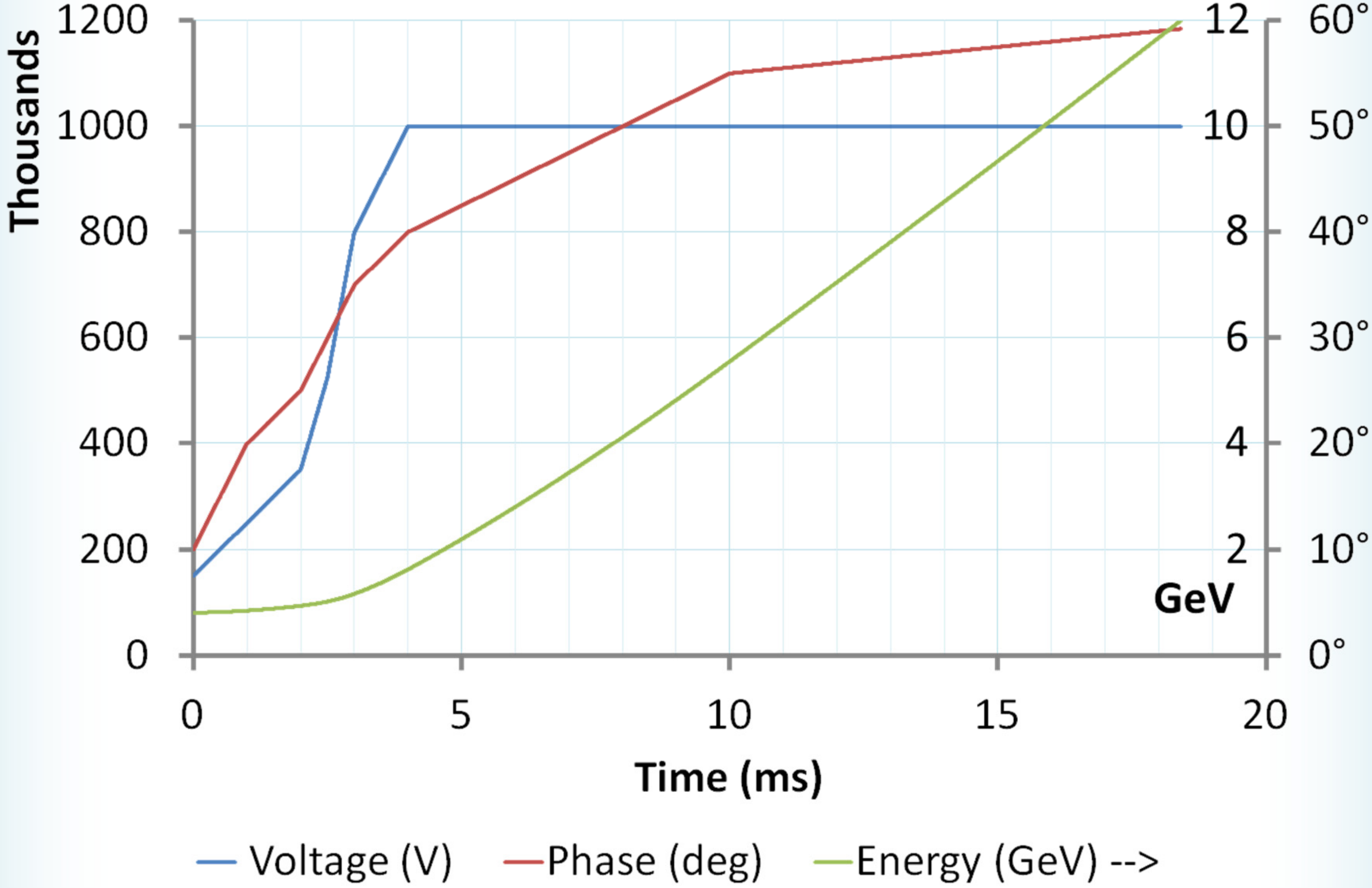
TABLE II. Longitudinal parameters for the 12 GeV VFFAG. Peak voltage per turn and phase are linearly interpolated from the times given.





RF harmonic	$h = 8$	
RF frequency	6.179–7.321 MHz	
Cycle duration	18.41 ms	
Rep. rate	50 Hz	
Time (ms)	**Voltage (kV)**	**Phase**
0	150	10°
1	250	20°
2	350	25°
2.5	525	30°
3	800	35°
4	1000	40°
10	1000	55°
18.41 (extract)	*1000*	*59.21°*
20	1000	60°

12GeV VFFAG RF Programme



Longitudinal Intensity Effects

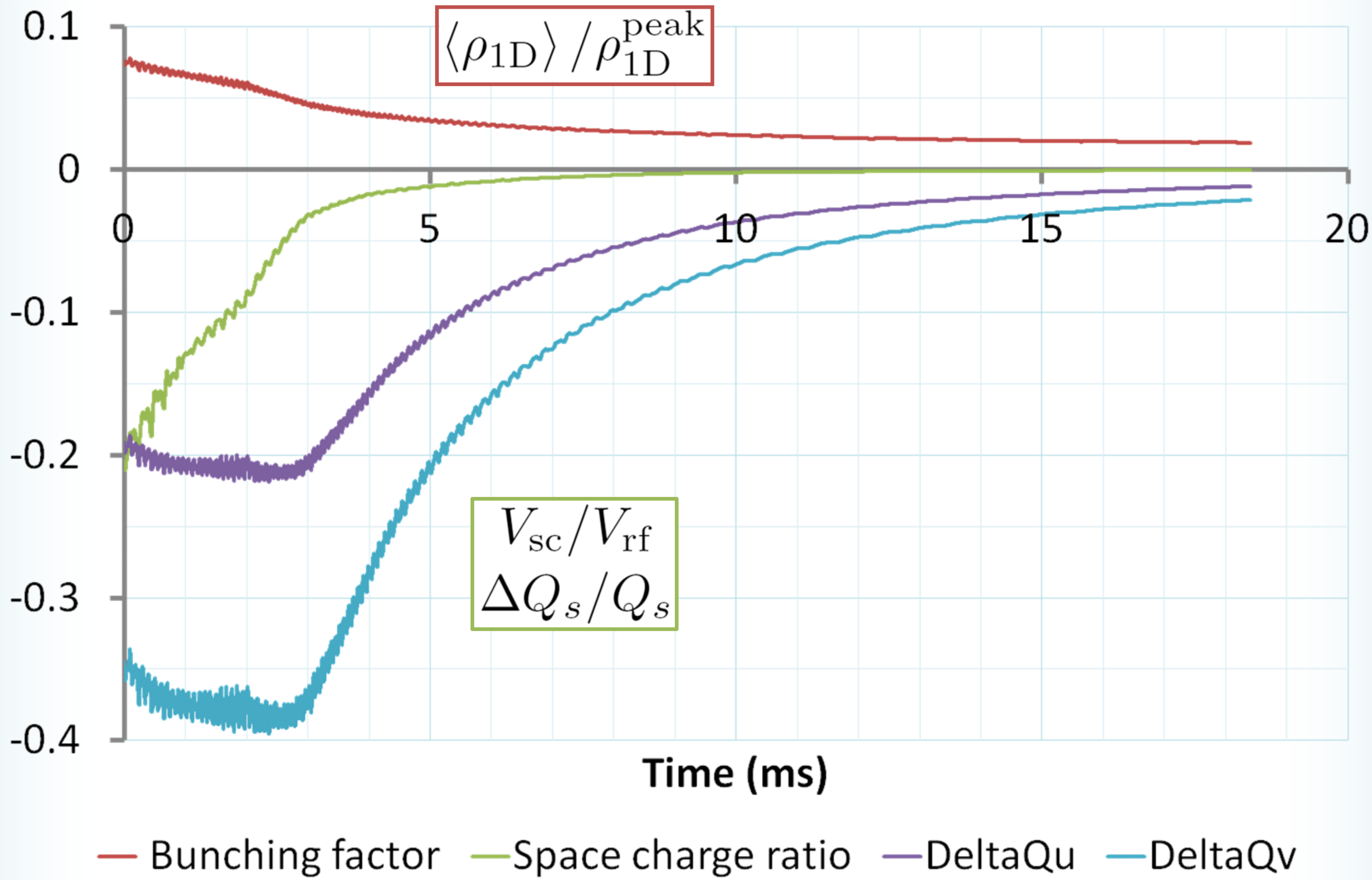


TABLE III. Intensity-dependent parameters for the ISIS single harmonic and 12 GeV VFFAG simulations run in series, for different numbers of protons injected into ISIS.

ISIS Protons In	**2.50e13**	**2.75e13**	**3.00e13**
ISIS μA in	200.3	220.3	240.3
ISIS transmission	90.54%	87.95%	85.98%
ISIS protons out	2.26e13	2.42e13	2.58e13
ISIS μA out	181.3	193.7	206.6
ISIS power (kW)	145	155	165
VFFAG transmission		100%	
VFFAG power (MW)	2.18	2.32	2.48
ISIS Peak Intensities			
Bunching factor	0.154	0.150	0.151
Space charge ratio	-0.301	-0.305	-0.311
$\Delta Q_{x,y}$	-0.499	-0.544	-0.580
VFFAG Peak Intensities			
Bunching factor	0.0188	0.0190	0.0190
Space charge ratio	-0.211	-0.257	-0.278
ΔQ_u	-0.219	-0.240	-0.254
ΔQ_v	-0.395	-0.434	-0.458

III. Isochronous 3D Cyclotrons

New: first successful tracking May 16th, 32 days ago

Isochronous Cyclotron Disease

- Mean radius must satisfy $r = \beta R$
  - Where $R = c/2\pi f_{rev}$ is the limiting radius as $v \rightarrow c$
- Mean $B_y = p/qr = m\beta\gamma c/q\beta R = \gamma(mc/qR) = \gamma B_0$
- This produces a quadrupole as radii bunch up:
  - $dB_y/dr = (B_0/R)d\gamma/d\beta = (B_0/R)\beta\gamma^3$
- Momentum only increases with $\beta\gamma$
  - Eventually quadrupole overfocusses the beam
  - Energy limit for any given planar cyclotron

Tilted Orbit Excursion

- Any angle θ is allowed, not just vertical!
  - Quadrupole field will rotate by θ/2
- Curved orbit excursion allows orbit radius ∝ velocity

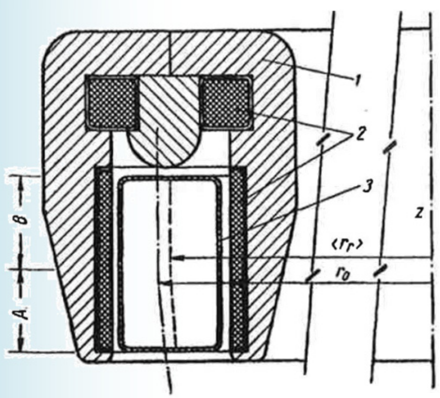
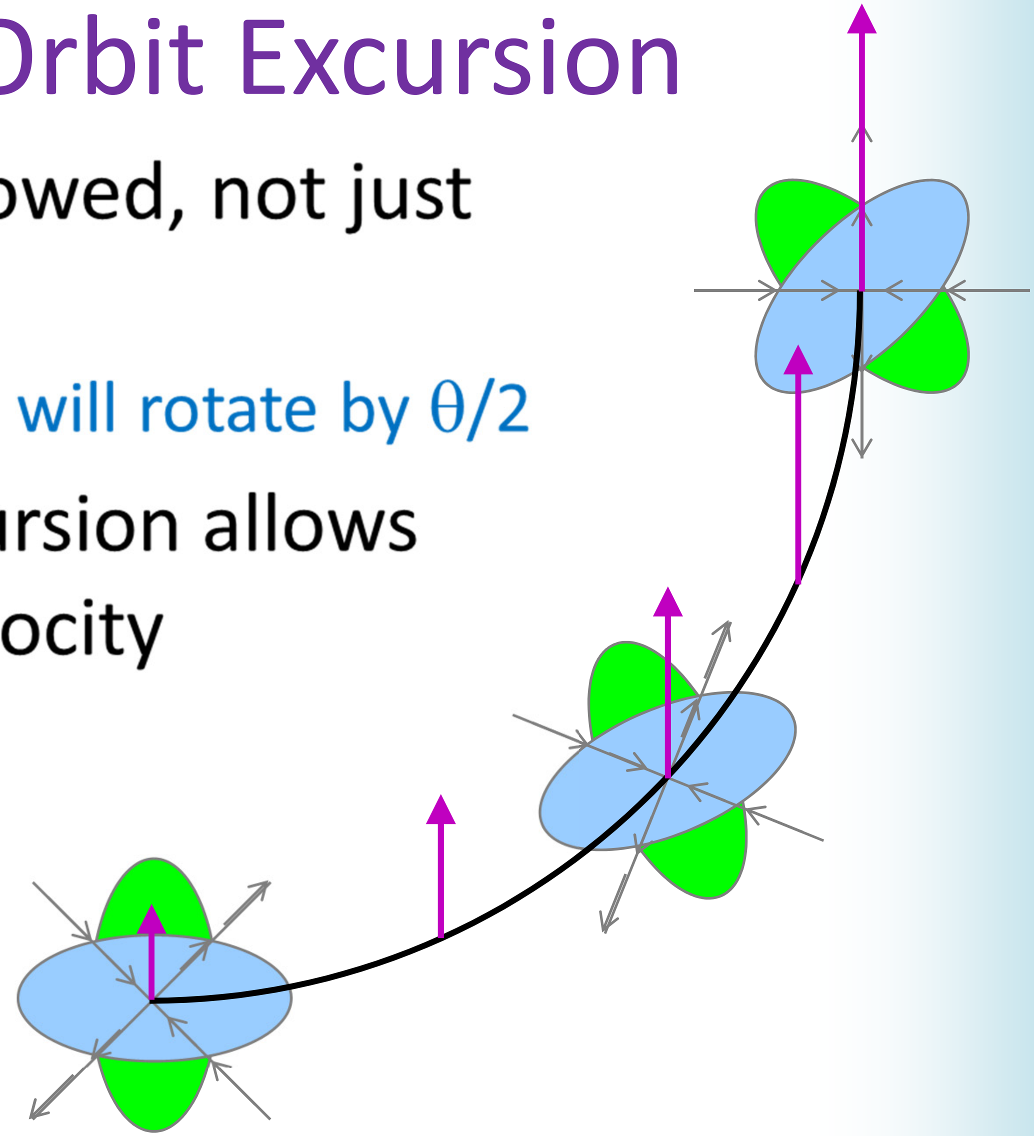


Fig. 1. Schematic section of accelerator with vertically increasing field: 1) ring magnet; 2) excitor windings for directing and focusing fields; 3) vacuum chamber; A) relativistic region; B) ultrarelativistic region.



← Teichmann (1962) also had idea

Extrapolation from Curved Surface

- Define **C**(x,y,z)=**B**(x,Y(x,z)+y,z) for a reference surface y=Y(x,z) so that **C**(x,0,z) is the initial condition. Transforming Maxwell's equations in free space to act on **C** gives:

$$\partial_y \mathbf{C} = \begin{bmatrix} 1 & Y_x & 0 \\ -Y_x & 1 & -Y_z \\ 0 & Y_z & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 & \partial_x & 0 \\ -\partial_x & 0 & -\partial_z \\ 0 & \partial_z & 0 \end{bmatrix} \mathbf{C}$$

$$\partial_z C_x - \partial_x C_z = Y_z \partial_x C_y - Y_x \partial_z C_y$$

$$\mathbf{B}_N(x,y,z) = \sum_{n=0}^{N} \frac{(y - Y(x,z))^n}{n!} \partial_y^n \mathbf{C}(x,0,z)$$

3D Cyclotron Field Model

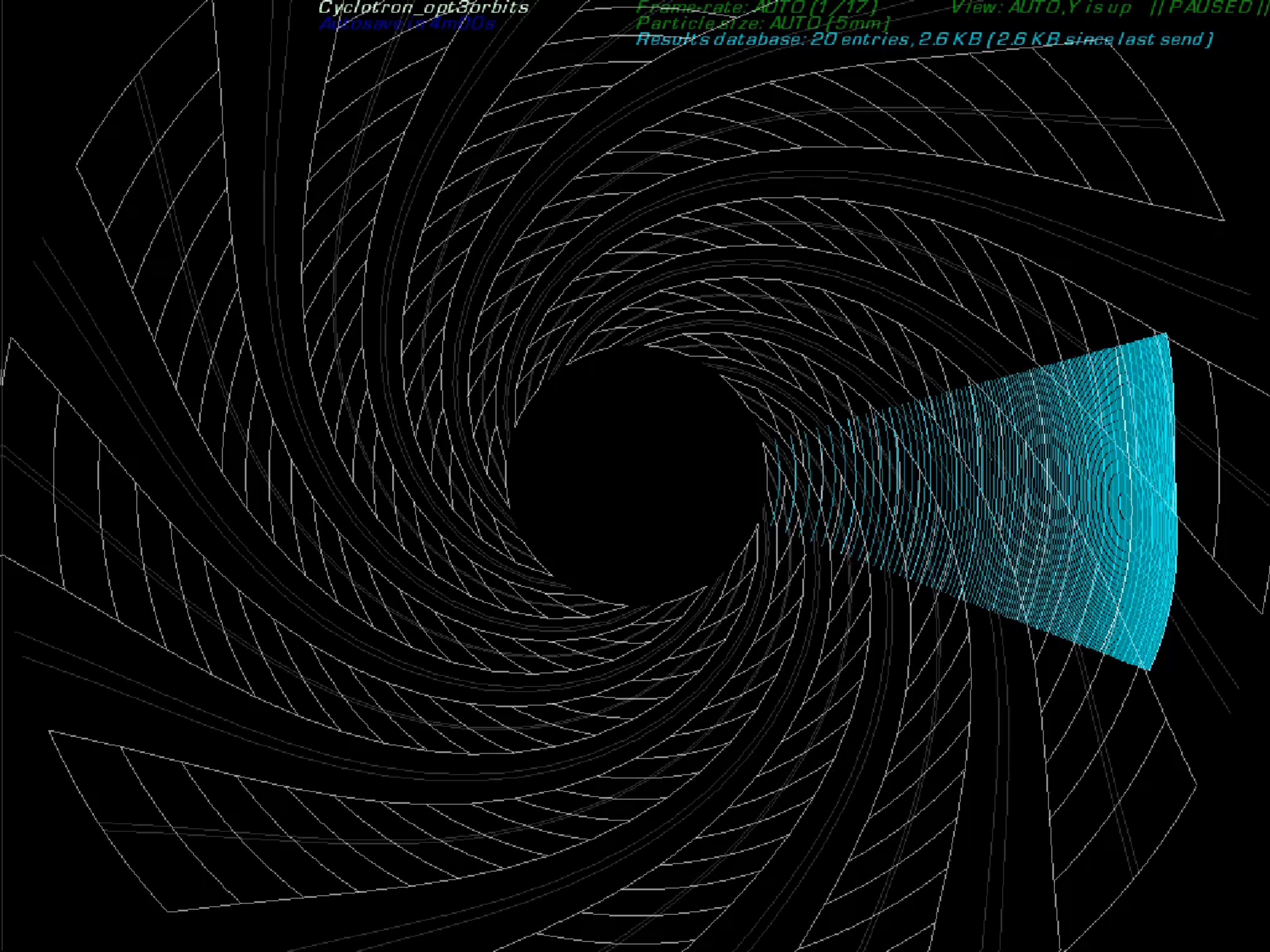
- Spiral angular coordinate $\eta = \theta - (\tan\theta_e)\ln r$
- Isochronous sector field form:

$$B_y(x, Y(x,z), z) = B_0 \gamma g(\eta) = \frac{B_0}{\sqrt{1-(r/R)^2}} g(\eta)$$

- Must satisfy:

$$B_0 = mc/qR \qquad\qquad \langle g(\eta)\rangle = 1$$

- Sectors constructed using $\pm\tanh((\eta - \eta_n)/\theta_f)$


Cyclotron_opt3orbits
Autosave in 4m00s
Frame-rate: AUTO (1/17)
View: AUTO, Y is up || PAUSED ||
Particle size: AUTO (5mm)
Results database: 20 entries, 2.6 KB (2.6 KB since last send)

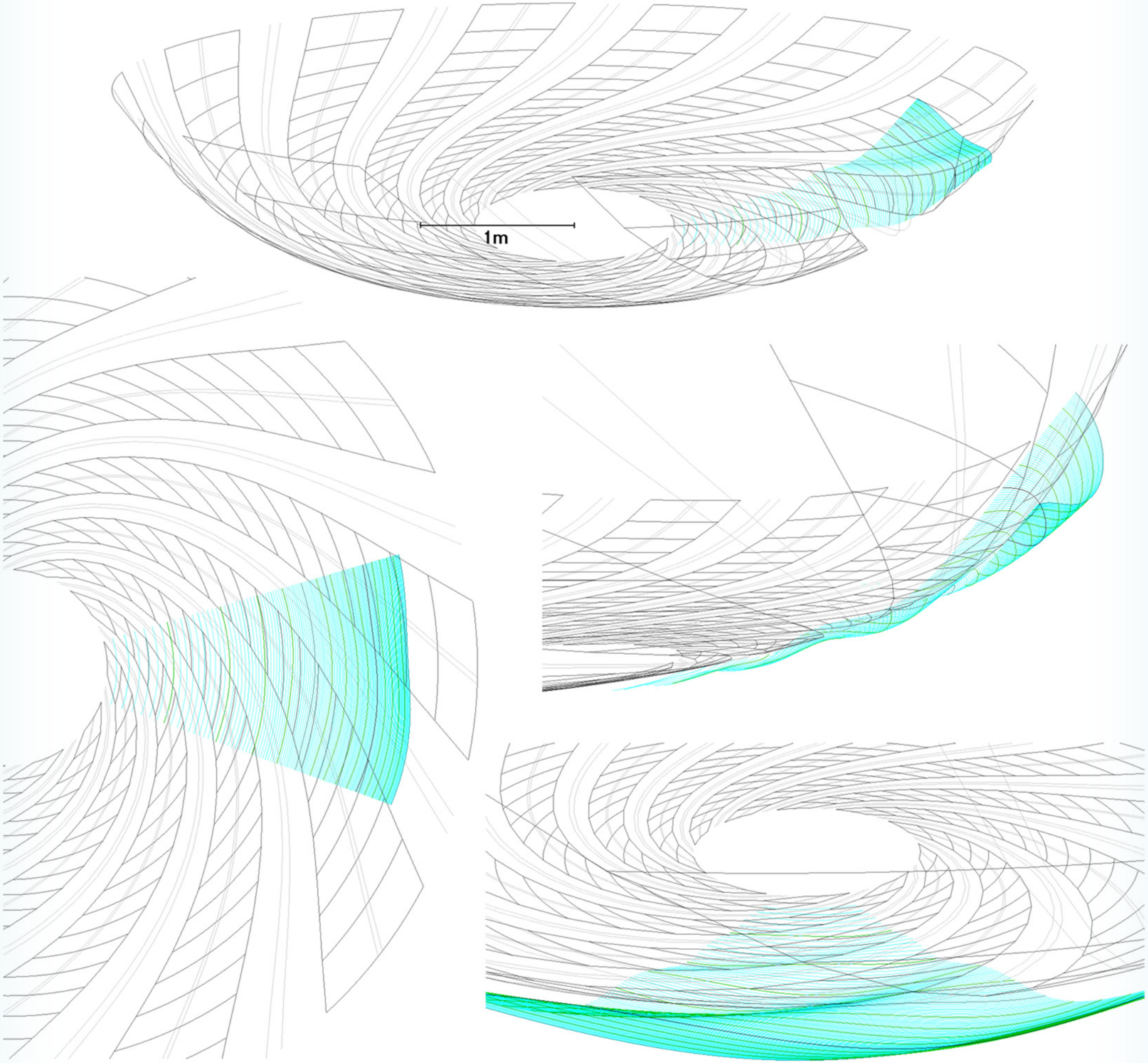
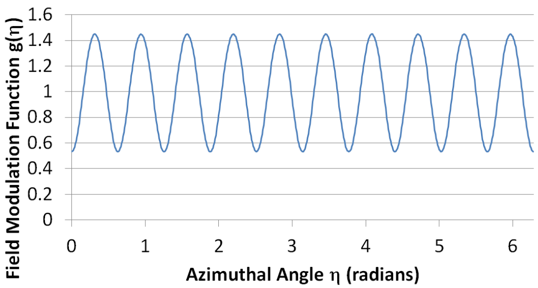
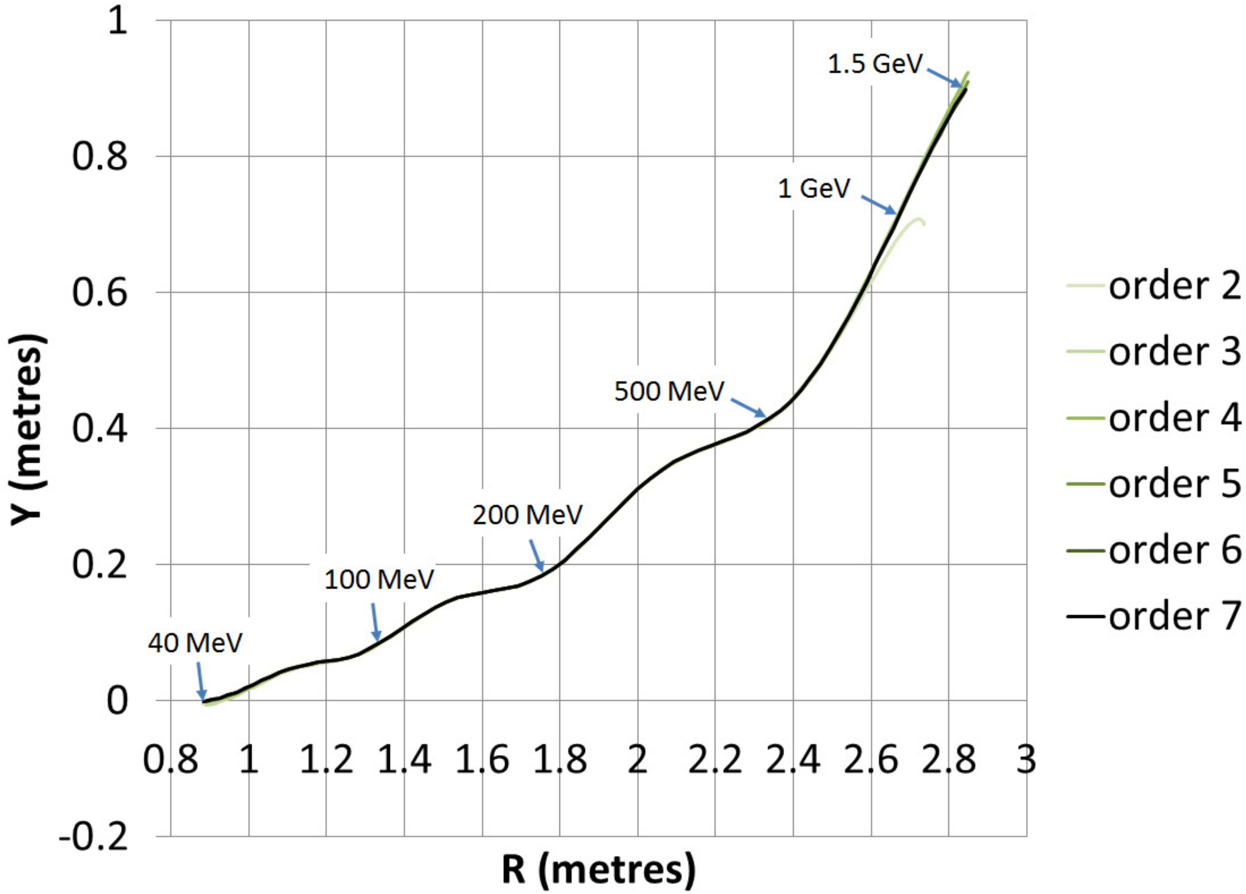

1m

Table 1: Parameters of the 3D Cyclotron

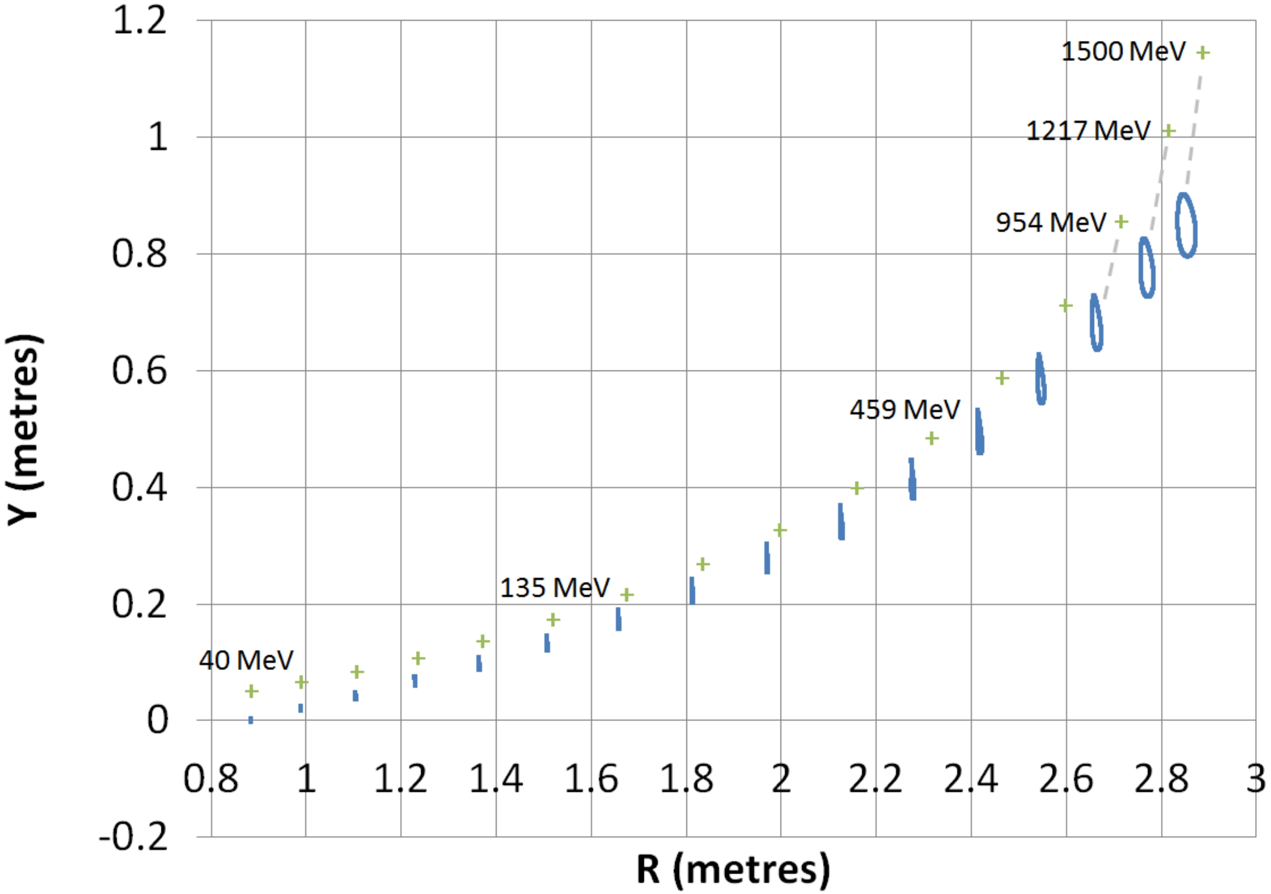
Energy range	40–1500	MeV
Radius range	0.8833–2.8738	m
Height range	−0.0023–0.9017	m
Maximum field on orbit	6.747	T
Revolution frequency	15.364±0.096	MHz
Sectors	10	
Sector edge angle θ_e	−63.43	°
Packing factor	54.35	%
Fringe extent θ_f	9.35	°
Mean field (γ=1) B_0	−1	T
Asymptotic radius R	3.1297	m
Reference height $Y(\beta) = 0.5324\beta^2 + 1.3168\beta^4 - 2.7235\beta^6 + 2.6954\beta^8$		m



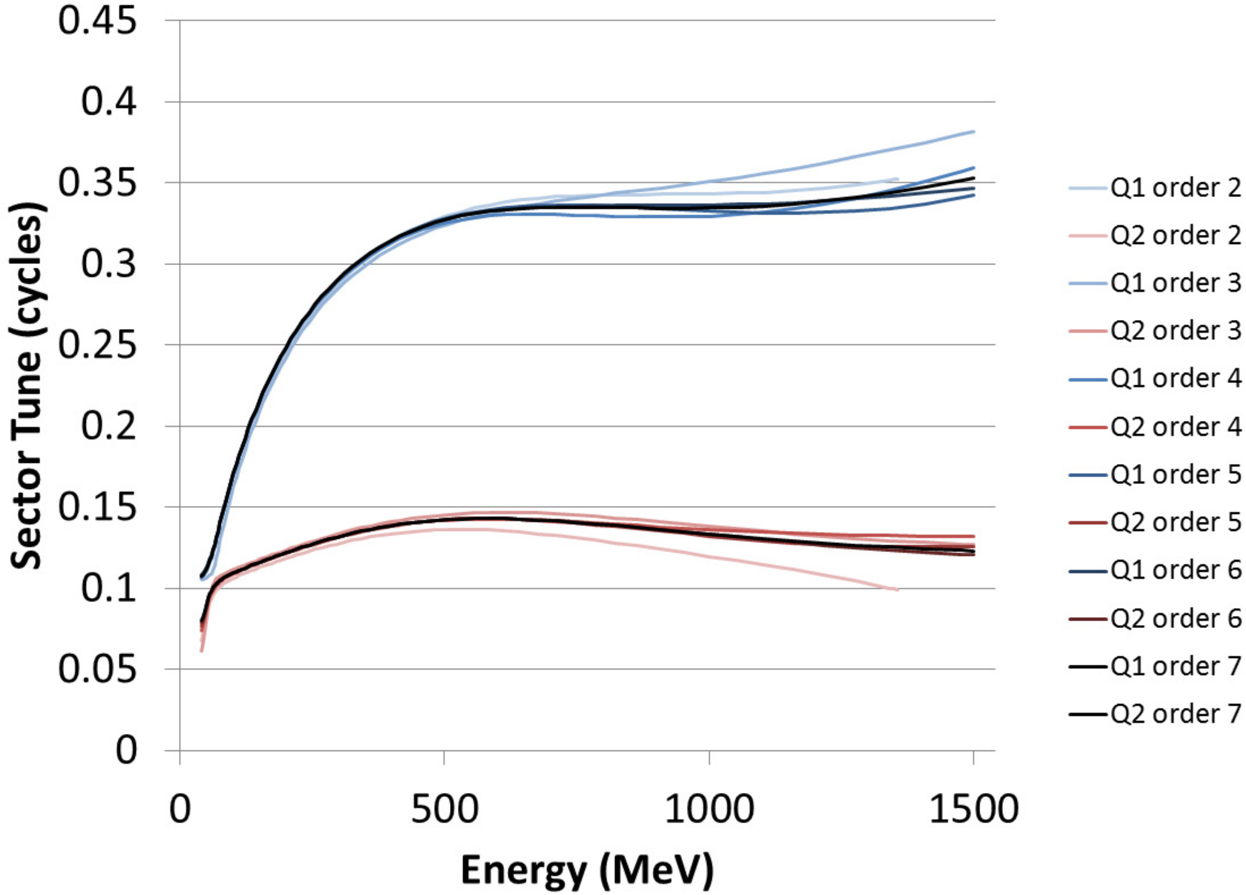
Orbit Locations at Matching Plane



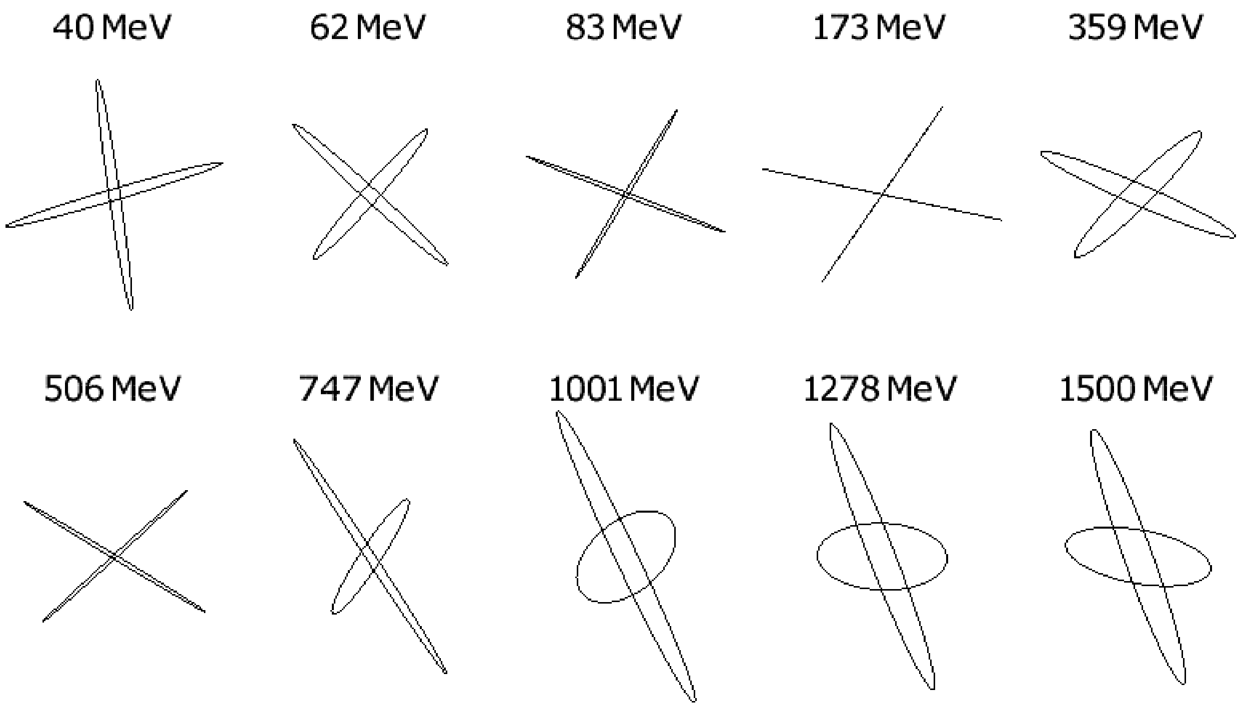
Deviation from Theoretical Orbit



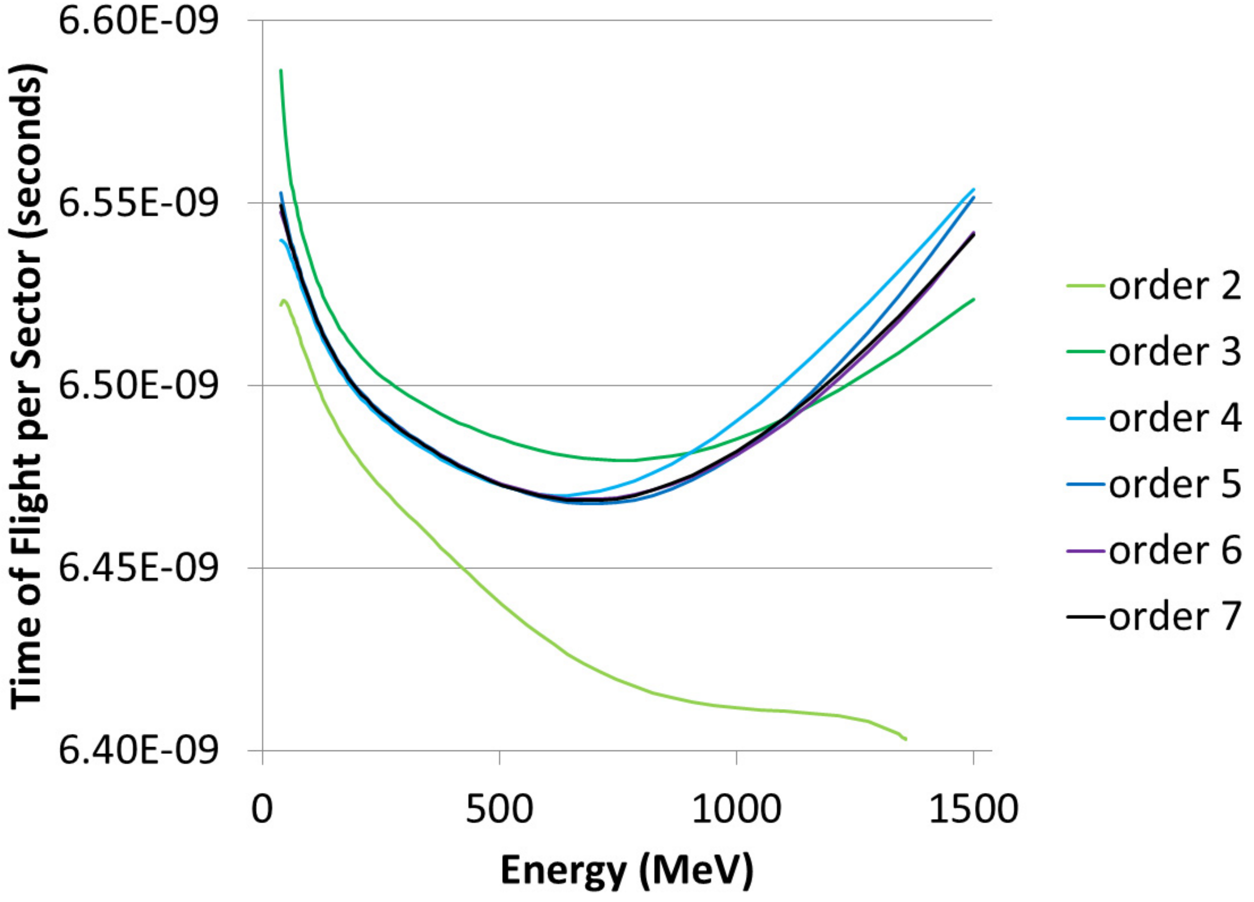
Eigentunes Stable at High Energy



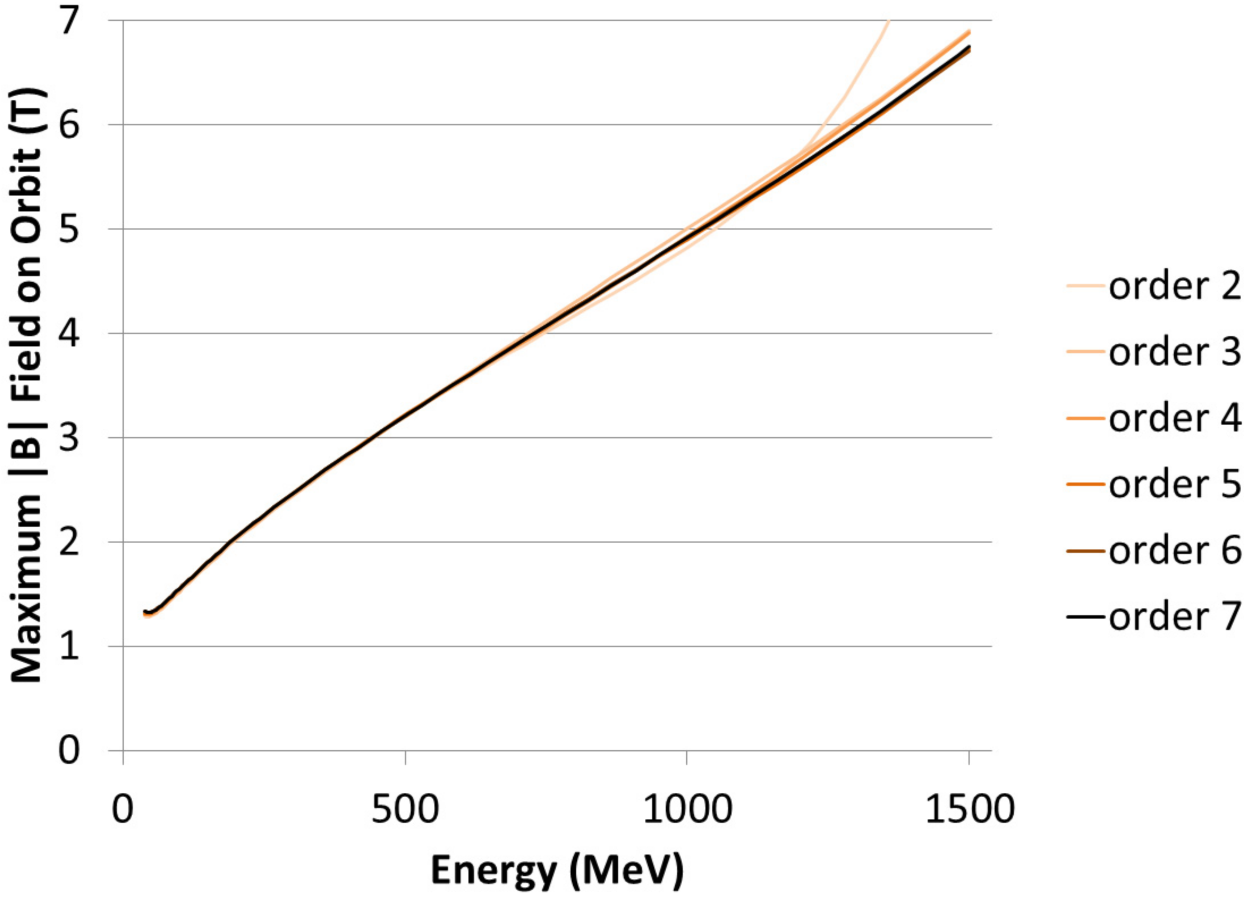
Focussing Eigenplanes in X-Y Space



Isochronism ±0.62%



Fields on Orbits <7T for R=3.1m



Comparison with Planar Cyclotron

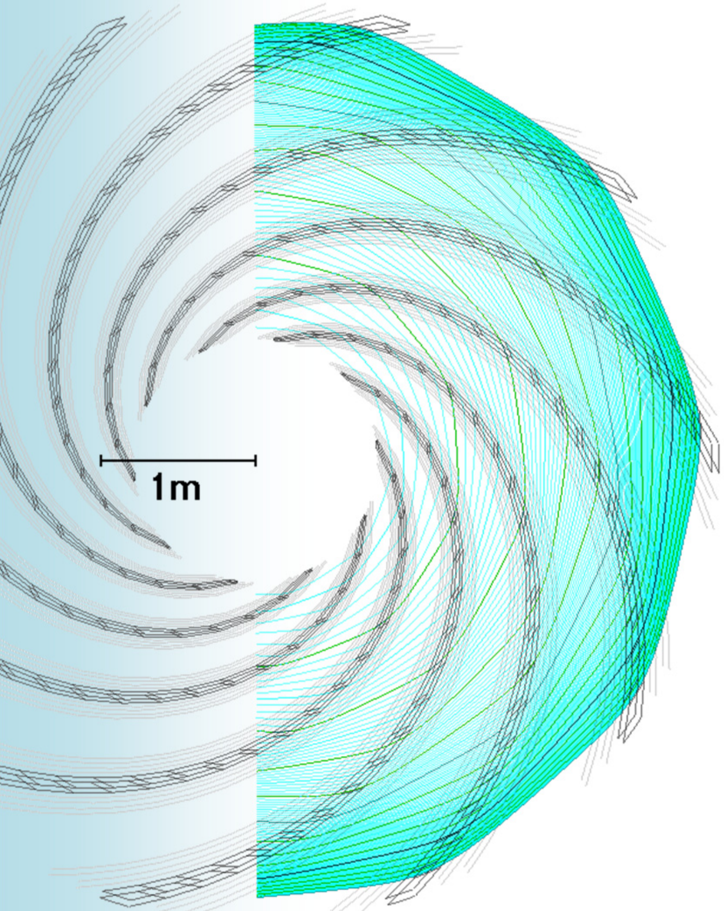
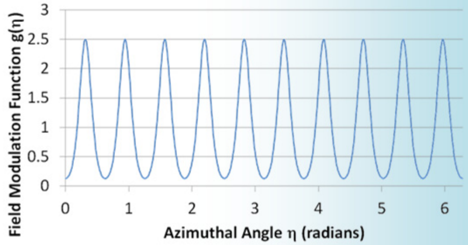
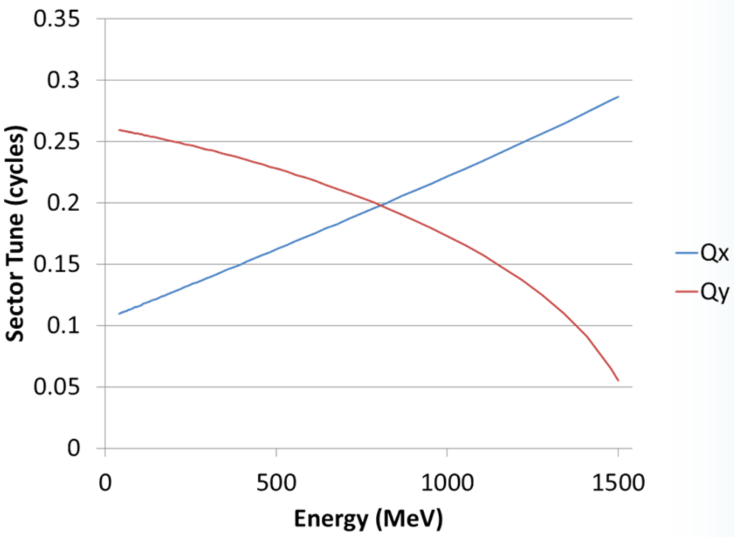


Table 2: Parameters of the Planar Cyclotron

Energy range	40–1500	MeV
Radius range	0.8684–2.9032	m
Maximum field on orbit	6.690	T
Revolution frequency	15.323±0.017	MHz
Sectors	10	
Sector edge angle θ_e	−63.43	°
Packing factor	10.21	%
Fringe extent θ_f	7.04	°
Mean field (γ=1) B_0	−1	T
Asymptotic radius R	3.1297	m





Conclusion & Applications

- Non-isochronous proton VFFAGs provide rapid cycling, high fields and compact SC magnet designs
  - Neutron, neutrino and hadron therapy sources
- Isochronous proton 3D cyclotrons promise cyclotron-like CW acceleration above 1GeV
  - High cross sections for nuclear/spallation reactions
  - Nuclear waste transmutation, isotope production
- Electron VFFAGs are already almost isochronous
  - Multi-pass VFFAG-ERL-FELs with only one recirculating arc

Future Work

- Better alignment between the reference plane and the orbits, giving better convergence
- Field adjustments to improve isochronism from the current ±0.62% variation
- Vary θ_e with energy to improve tune control
- Make space for RF by using fewer sectors and/or more field-free space between sectors
- Find an example superconducting winding scheme
- Build electron model of the 3D cyclotron