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SIMULATION STUDY ON BEAM LOSS IN THE ALPHA BUCKET REGIME DURING SIS-100 PROTON OPERATION

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Abstract

Besides heavy ion operation, the heavy ion synchrotron SIS-100 will accelerate a single proton bunch of $2 \cdot 10^{13}$ particles up to the energy $E = 29$ GeV. For the present standard scenario, optics settings have been developed which provide a transition energy according to $\gamma_{tr} = 45.5$ in order to avoid transition crossing during acceleration. At extraction energy the corresponding nonlinear momentum compaction and phase slip factors cause the formation of a so called alpha bucket. In this contribution we present the results of transverse beam loss tracking studies in the alpha bucket regime. The effects of momentum spread, magnet errors and residual closed orbit distortion are analyzed.

THE SIS-100 PROTON CYCLE

The future heavy ion synchrotron SIS-100 is projected to accelerate heavy ion and proton beams of high intensity. In the present standard proton scenario four bunches each with $5 \cdot 10^{12}$ protons will be injected at $E_{ini} = 4$ GeV, merged to a single compressed bunch, and accelerated to the final energy $E_{fin} = 29$ GeV. A proton cycle will start with optics which creates $\gamma_{tr,ini} = 18.3$. During the ramp γ_{tr} will be increased to its final value $\gamma_{tr,fin} = 45.5$ to keep it above γ and to avoid transition crossing. In doing so, the strongly increased maximum values of horizontal beta and dispersion functions, $\beta_{x,y,max}$ and $D_{x,max}$, and chromaticities $\xi_{x,y}$ due to the high-γ_{tr} optics occur only at high energies, when the transverse emittances are small, see Table 1. As a result,

Table 1: Lattice and beam parameters during SIS-100 proton operation assumed in the simulations. The natural chromaticities $\xi_{x,nat}, \xi_{y,nat}$ correspond to Eq. (1).

	Injection	Extraction
$E/(\mathrm{GeV})$	4 GeV	29 GeV
(Q_x, Q_y)	$(21.8, 17.7)$	$(21.8, 17.7)$
γ_{tr}	18.3	45.5
$(\epsilon_x, \epsilon_y)_{rms}$	$(3.2, 1.0)$ μm	$(0.53, 0.16)$ μm
$(\beta_x, \beta_y)_{max}$	$(19.2, 20.9)$ m	$(71.3, 29.7)$ m
$D_{x,max}$	1.3 m	2.9 m
$(\xi_x, \xi_y)_{nat}$	$(-1.24, -1.34)$	$(-2.43, -1.37)$
V_{rf}	20 kV	300 kV

the maximum chromatic tune deviations

$$\Delta Q_{z,max} = \xi_z Q_z \delta_{max}, \quad z = x, y, \tag{1}$$

at extraction energy are large and the horizontal chromatic tune spread even exceeds 0.5, see Fig. 1. Besides, the lowest

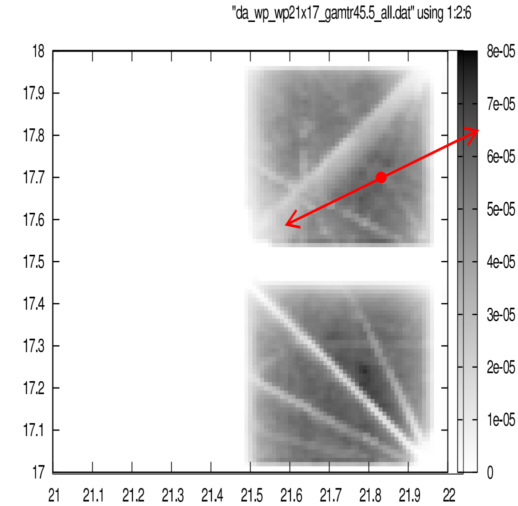

Figure 1: Dynamic aperture scan for extraction conditions and systematic and random multipole errors in dipole and quadrupole fields to make resonances visible. The red double arrow denotes the tune spread due to the natural chromaticities in Table 1 and the symmetric, numerically calculated momentum spread $\delta_{max} = \pm 0.0043$ [1].

order momentum compaction and phase slip factors in the high-γ_{tr} optics are very small: $\alpha_0 = 1/\gamma_{tr}^2 = 4.8 \cdot 10^{-4}$ and $\eta_0 = \alpha_0 - 1/\gamma^2 = -5.0 \cdot 10^{-4}$. Hence, contributions of higher order in δ have strong influence and an alpha bucket with the unstable fixed point $(0, \delta_{UFP})$ is created, see Fig. 2. The focus of this work is on consequences of the alpha bucket. Hence, only extraction conditions are considered. Effects due to acceleration are neglected, magnet field errors and misalignments leading to resonance excitation and closed orbit deformation are regarded. For all simulations MAD-X was used [2].

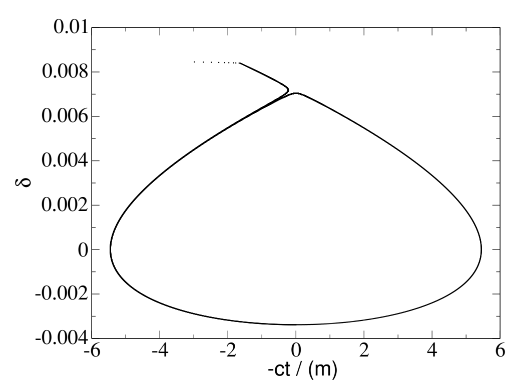

Figure 2: Stable and unstable particle trajectories with $\epsilon_x = \epsilon_y = 0$ in an alpha bucket for natural chromaticities. The unstable fixed point is at $-ct_{UFP} = 0$, $\delta_{UFP} = 0.0071$.

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CHROMATICITY CORRECTION

Fig. 1 shows that the chromaticities have to be reduced to make the chromatic tune spread fit in the interval $Q_x \in [21.5, 22]$. Actually, the reduction of the maximum chromatic tune deviations to $\Delta Q_{x,max} = \Delta Q_{y,max} = \pm 0.05$ was found to be proper. Using $\delta_{max} = \pm 0.0043$ [1] the chromaticities have to be $\xi_{x,corr} = -0.53$, $\xi_{y,corr} = -0.66$. To enhance the chromaticity correction efficiency, a sextupole powering scheme was developed which satisfies

$$\sum_n (k_2 L)_n^2 \left(\beta_{x,n}^2 - \kappa D_{x,n}^2\right) \to \text{min.}, \tag{2}$$

where $\kappa = 0, 1$ and $|D_{x,n}| < \beta_n$ in all sextupole positions s_n. In doing so, the impact of the sextupoles as non-linear lenses is reduced by minimising their strengths in positions of large horizontal beta function, whereas the chromaticity correction impact is increased by maximising their strengths, where the horizontal dispersion is large. The formulas for the corrected chromaticities [3],

$$\xi_{z,corr} = \xi_{z,nat} \pm \frac{1}{4\pi Q_z} \sum_n (k_2 L)_n \beta_{z,n} D_{x,n},\ z = x, y, \tag{3}$$

were added as constraints. Plus and minus correspond to horizontal and vertical chromaticities, respectively.

In addition, the momentum of the unstable fixed point is increased by correcting the chromaticities from $\delta_{UFP} = 0.0071$ to $\delta_{UFP} = 0.013$ leading to a larger bucket area.

CLOSED ORBIT AND MAGNET ERRORS

The amount of lost particles depends on resonance excitation and closed orbit deformation due to magnet field errors and misalignments. Five different random samples of magnet field errors and misalignments were used.

Random transverse shifts of the quadrupoles, $\Delta x_{quad}, \Delta y_{quad}$, and rotations of the dipoles around the z axis, $\Delta\psi_{dip}$, as well as random deviations of the bending angles of the dipoles, $\Delta\alpha_{bend}$, had been identified as major sources of closed orbit deformations in former studies and were regarded in this study. They were modelled by Gaussian distributions truncated at 2σ with

$$\sigma_{\Delta x, \Delta y, quad} = 1.0 \text{ mm} \tag{4}$$

$$\sigma_{\Delta\psi, dip} = 1.43 \text{ mrad} \tag{5}$$

$$\frac{\sigma_{\Delta\alpha_{bend}}}{\alpha_{bend}} = 0.004. \tag{6}$$

The resulting closed orbit deviations were corrected applying the MICADO algorithm implemented in MAD-X to all 84 horizontal and vertical steerers. The average residual rms closed orbit deviations of all random error samples applied are $x_{co,rms} = 1.3$ mm and $y_{co,rms} = 0.36$ mm.

Resonance excitation was modelled by introducing systematic and random magnet field errors represented by normal and skew multipoles of orders $0, \ldots, 15$ in dipoles and quadrupoles. The systematic multipoles are results of magnet field simulations [4, 5]. The random multipoles were generated according to Gaussian distributions truncated at 2σ with σ for each order set to 30 % of the systematic normal multipole of the same order.

INITIAL PARTICLE DISTRIBUTION

In order to minimise artificial alteration of the computed particle losses the initial distribution of the test particle coordinates has to be matched. The transverse initial coordinates were generated centered around the closed orbit according to Gaussian distributions truncated at 2σ with σ according to the the rms emittances in Table 1.

For generating the longitudinal initial coordinates the non-symmetric bucket and bunch shapes with respect to the sign of δ have to be regarded. Besides, the stable fixed point in the “centre” of the longitudinal phase space trajectory of a particle is shifted to a finite momentum δ_{SFP} if its emittances are finite or the closed orbit is deformed.

The momenta of the fixed points of synchrotron motion are determined by the zeros of the momentum dependent revolution time deviation, $\Delta T(\delta)/T_0 = 0$. Applying

$$\frac{\Delta T(\delta)}{T_0} = \chi + \eta_0 \delta + \eta_1 \delta^2 \tag{7}$$

yields the momenta of the unstable and stable fixed points

$$\delta_{UFP,SFP} = -\frac{\eta_0}{2\eta_1}\left(1 \pm \sqrt{1 + \frac{4\eta_1 \chi}{\eta_0^2}}\right). \tag{8}$$

$\chi = \chi_\epsilon + \chi_{co}$ represents the increase in single-turn path length due to finite emittances and closed orbit distortion [6] and does not depend on δ. Hence, it is the same for the increase in revolution time.

In the first step, δ_{SFP} was obtained starting with the determination of δ_{UFP} by tracking particles with $\epsilon_x = \epsilon_y = 0$ in the lattice without magnet field errors and misalignments but with the sextupoles set to correct the chromaticities. Inserting δ_{UFP} in the equation [3]

$$\delta_{UFP} = -\eta_0/\eta_1 \tag{9}$$

yielded η_1. Afterwards, the value of δ_{SFP} for $\chi_\epsilon = 0$ and the actual sample of magnet field errors and misalignment was extracted from MAD-X Twiss and inserted in Eq. (8) in order to find χ_{co}. χ_ϵ was calculated using

$$\chi_\epsilon = \frac{1}{4}\left(\epsilon_x \langle\gamma_x\rangle \frac{\xi_{x,corr}}{\xi_{x,nat}} + \epsilon_y \langle\gamma_y\rangle \frac{\xi_{y,corr}}{\xi_{y,nat}}\right), \tag{10}$$

where $\gamma_z = (1 + \alpha_z)/\beta_z$, $z = x, y$, and $\langle \ldots \rangle$ denotes the average along the accelerator. Finally, η_0, η_1, and $\chi = \chi_\epsilon + \chi_{co}$ were inserted in Eq. (8) to find δ_{SFP}. Eq. (10) differs from the textbook formula in [6] by the ratios of corrected and natural chromaticities. They were found to make δ_{SFP} approximately fit the centres of numerically obtained synchrotron orbits, see Fig. 3.

The stable fixed point variable can be applied to create the longitudinal distribution using the ansatz

$$f(H) \propto \mathrm{e}^{-H(\phi,\delta)/H_{max}} \Theta\left[H_{max} - H(\phi, \delta)\right] \tag{11}$$

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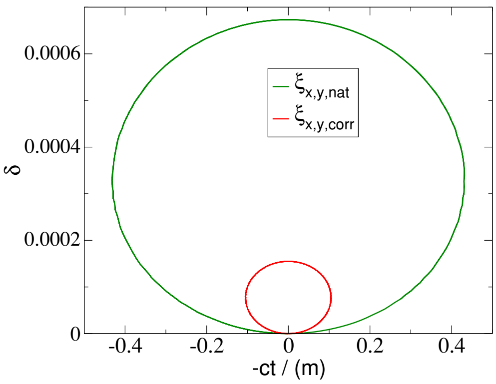

Figure 3: Synchrotron orbits of two particles, both with initial emittances $\epsilon_x = 1\ \mu$m$, \epsilon_y = 0$, and longitudinal coordinates $(-ct, \delta) = (0, 0)$. Magnet field errors and misalignments are not applied. Using the natural chromaticities, $\xi_{x,y,nat} = -2.43, -1.37$ the centre of the trajectory was found at $\delta_{centre} = 3.4 \cdot 10^{-4}$. Using the corrected chromaticities, $\xi_{x,y,corr} = -0.53, -0.66$, led to $\delta_{centre} = 7.7 \cdot 10^{-5}$. The corresponding stable fixed point momenta obtained with Eq. (8) are $\delta_{SFP} = 3.2 \cdot 10^{-4}$ and $\delta_{SFP} = 6.8 \cdot 10^{-5}$.

with the Heaviside function Θ and the maximum Hamiltonian H_{max} given by the bunch area. In small-amplitude-approximation the Hamiltonian without acceleration reads

$$H(\phi, \delta) = h\left(\chi\delta + \frac{\eta_0}{2}\delta^2 + \frac{\eta_1}{3}\delta^3 - \chi\delta_{SFP} - \frac{\eta_0}{2}\delta_{SFP}^2 - \frac{\eta_1}{3}\delta_{SFP}^3\right) - \frac{eV_{rf}}{4\pi\beta_0^2 E}\phi^2 \qquad (12)$$

which ensures that $H = 0$ for a particle at the stable fixed point and Eq. (7) can be found from H. By defining

$$\Delta^2 \equiv \delta^2 + \frac{2\chi}{\eta_0}\delta + \frac{2\eta_1}{3\eta_0}\delta^3 - \delta_{SFP}^2 - \frac{2\chi}{\eta_0}\delta_{SFP} - \frac{2\eta_1}{3\eta_0}\delta_{SFP}^3, \qquad (13)$$

where Δ^2 can be produced for each particle by a Gaussian random number generator, the corresponding δ can be calculated by applying Cardan's formula for solving third order equations. Out of the three solutions those are used which converge towards Δ for $\chi, \eta_1 \to 0$.

PARTICLE LOSS

In order to estimate beam loss at proton extraction energy, particle tracking simulations were performed, where 1000 test particles were tracked for 32000 turns what approximately corresponds to two synchrotron periods. Five different random samples of magnet field errors and misalignments were applied. The alteration of η_0 arising from errors in the focusing strengths of the main quadrupoles was regarded in the generation of the initial particle coordinates whereas $\delta_{UFP} = 0.13$ was kept for the determination of η_1 by Eq. (9) in all simulations. The resulting particle losses depend strongly on the samples of magnet errors, whereas the difference between $\kappa = 0, 1$ in the chromaticity correction scheme is low. Furthermore, larger particle loss was found for smaller γ_{tr}, see Table 2.

Table 2: γ_{tr} and relative particle loss P_{loss} found in simulations for $\kappa = 0, 1$ in Eq. (2) and the seed numbers applied to generate random magnet errors and misalignments.

n_{seed}	γ_{tr} $(\kappa = 0)$	P_{loss} $(\kappa = 0)$	γ_{tr} $(\kappa = 1)$	P_{loss} $(\kappa = 1)$
100	44.9	0.7 %	44.9	0.8 %
200	42.7	5.4 %	42.7	5.9 %
300	43.1	4.4 %	43.1	4.8 %
400	47.9	0.2 %	47.9	0.1 %
500	41.2	8.5 %	41.3	8.8 %
averaged	–	3.8 %	–	4.1 %

SUMMARY

The aim of the study presented in this paper was to estimate particle loss for extraction conditions of SIS-100 proton operation by particle tracking simulations using MAD-X. In order to reach the extraction energy without transition crossing, high-γ_{tr} optics are applied resulting in an alpha bucket and large chromaticities so that the horizontal chromatic tune spread exceeds 0.5. Therefore, proper settings for the sextupoles to reduce the chromaticities were developed and an initial particle distribution matched to the alpha bucket was generated. Particle loss simulations were performed for five samples of random magnet field errors and misalignments. Beam loss and γ_{tr} were found to strongly depend on the sample. An average beam loss of about 3 % has been found which is above the beam loss budget of 1 %. Furthermore, beam loss and γ_{tr} were found to be correlated what could help to find space for further optimisation in order to reduce the particle losses.

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