
Acceleration and Transportation of Multiple Ion Species at EBIS-Based Preinjector

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Acknowledgements



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- Brookhaven Accelerator Complex
- Overview of the EBIS Pre-injector
- Electron Beam Ion Source (EBIS)
- Why Multiple Ion Species
- Acceleration and Transportation of multiple ion species
- Conclusions

Many (or all) of these can be running simultaneously

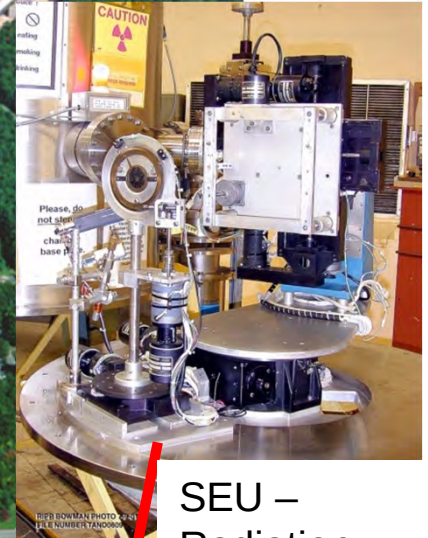
Heavy ions for the Single-Event Upset Facility



NSRL – biological effects of space radiation



EBIS



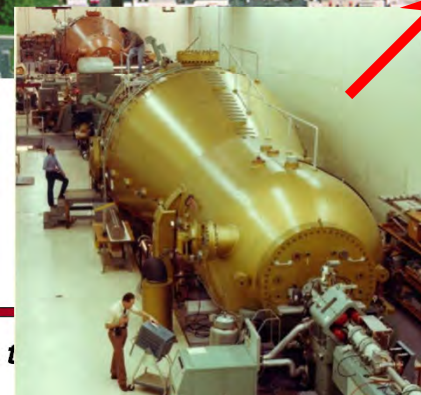
SEU – Radiation effects on electronics



200 MeV Linac



BLIP – medical isotopes

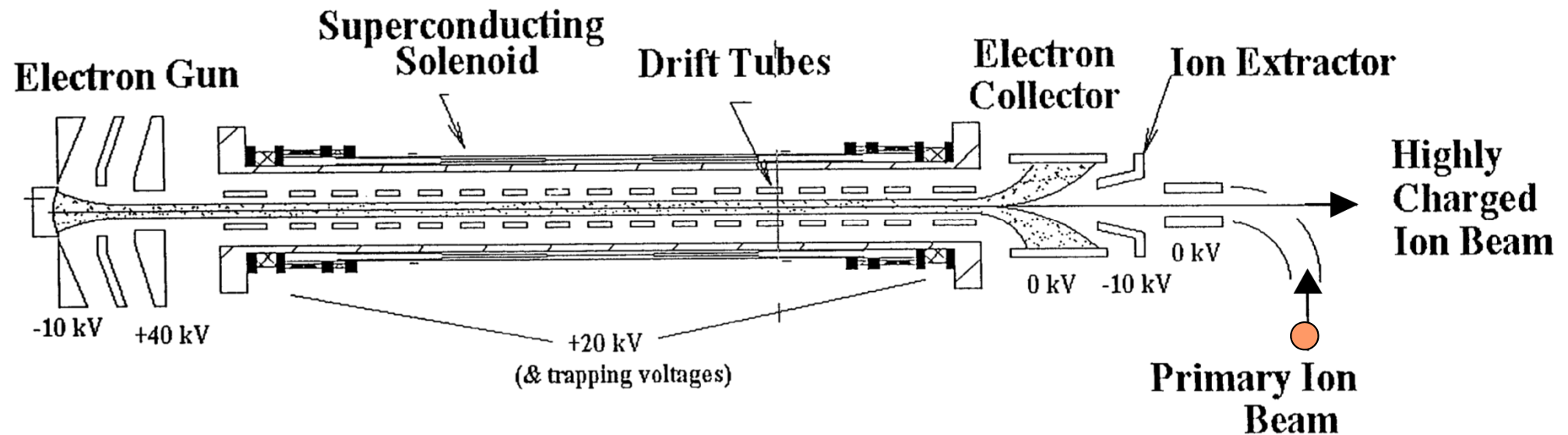


Two 14 MV Tandem Van de Graaffs

HB2012
Beijing, China

- Increased flexibility to handle the simultaneous needs of RHIC and NASA (fast switching between species ~ 1 s)
- Capability to provide ions not presently available, such as noble gas ions (for NASA), uranium (RHIC).
- Simpler technology, robust, more modern
- Elimination of two stripping stages and an 860 m long transport line, leading to improved performance

Principle of EBIS

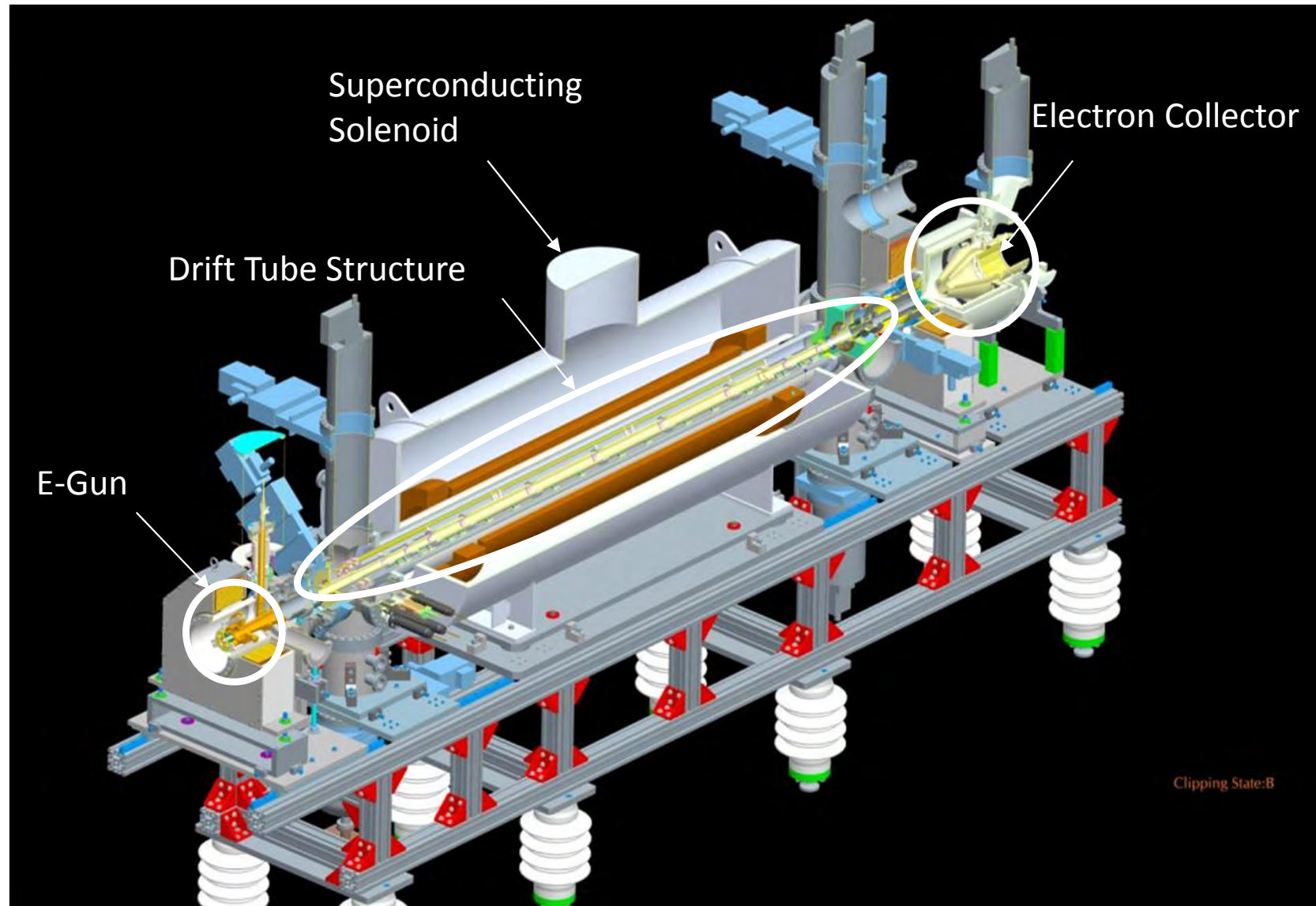


Radial trapping of ions by the space charge of the electron beam.
Axial trapping by applied electrostatic potentials at ends of trap.

- The total charge of ions extracted per pulse is $\sim (0.5 - 0.8) \times (\text{\# electrons in the trap})$
- Ion output per pulse is proportional to the trap length and electron current.
- Ion charge state increases with increasing confinement time.
- Output current pulse is $\sim$ independent of species or charge state!

Electron Beam Ion Source (EBIS)

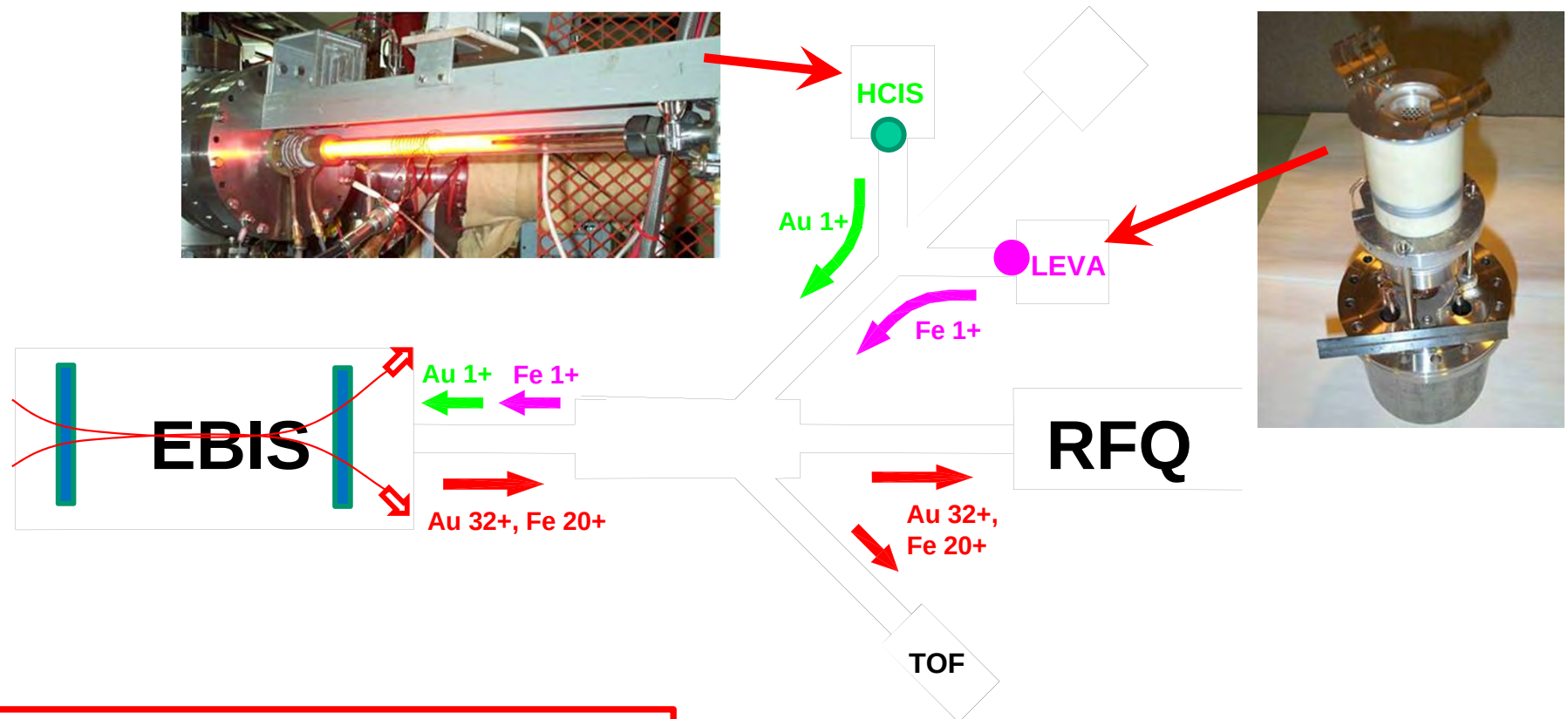
High charge state heavy ions produced by trapping and stepwise ionizing in an electron beam



BNL: 10 A electron beam; 5T magnet; 1.5 m long trap region

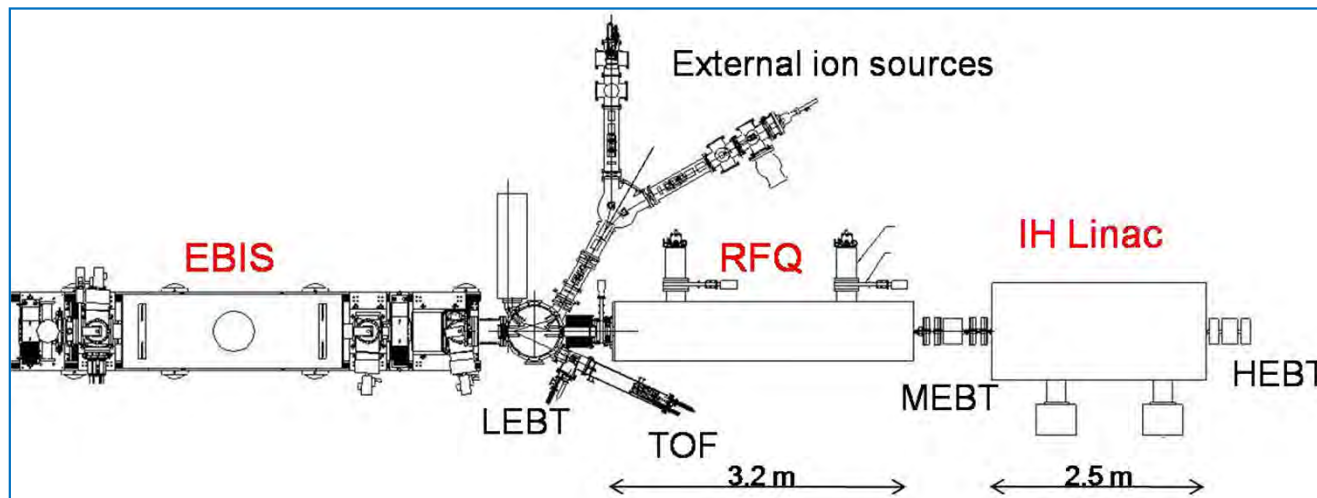
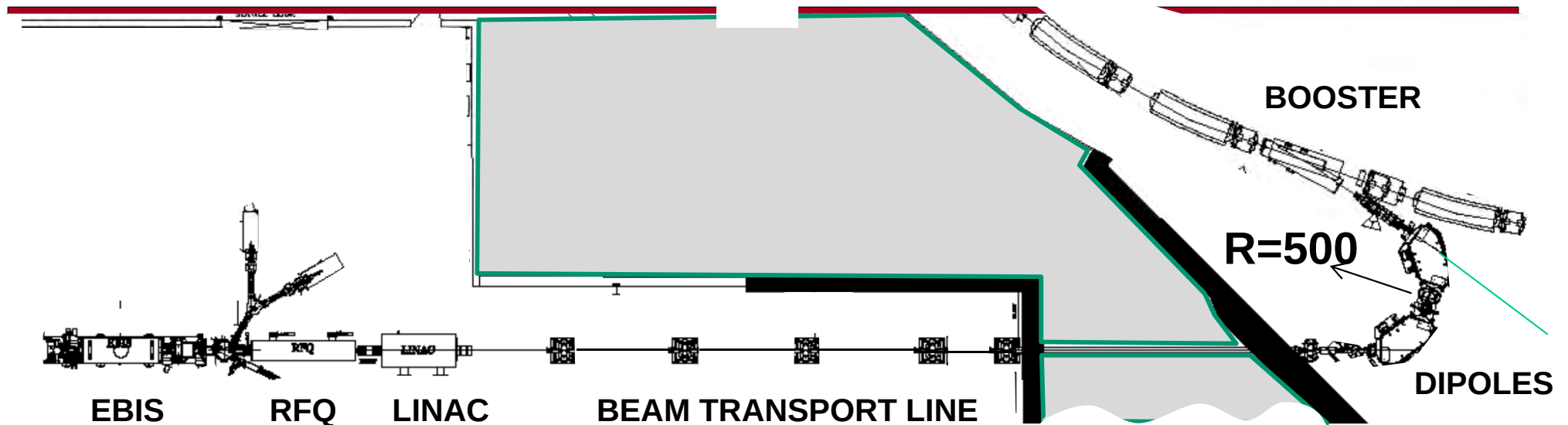
Ion Injection and Extraction from the EBIS

External ion injection provides most ion species.



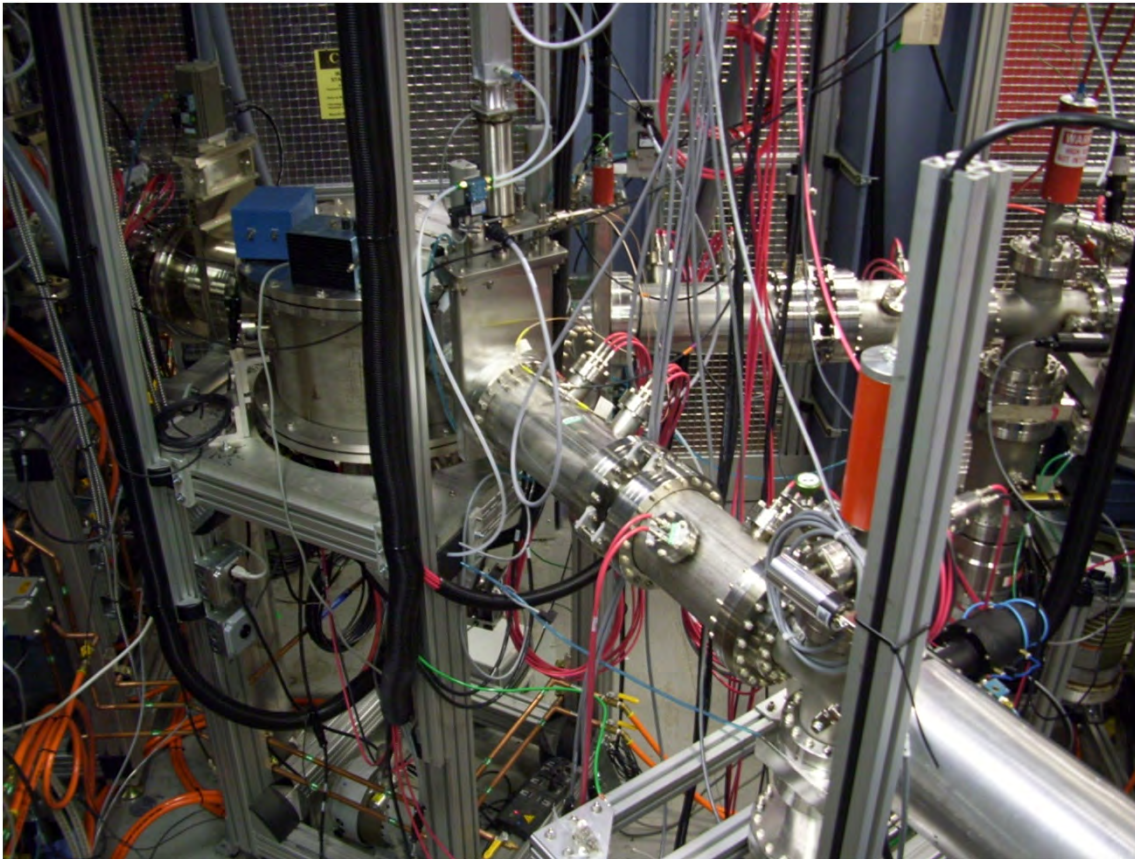
One can change species and charge state
on a pulse to pulse basis

EBIS Preinjector in lower equipment bay of the 200 MeV Linac



Ions	He - U
Q / m	$\geq 1/6$
Current	$> 1.5 \text{ emA (10 } \mu\text{S)}$
Pulse length	10-40 μs
Rep rate	5 Hz
Output energy	2 MeV / u
$\Delta P/P$	$< 0.05\%$
Time to switch species	1 second

1+ ion injection lines



Low Energy Vacuum Arc
Source (I. Brown);



Hollow Cathode Ion Source
based on design used on Saclay
EBIS.



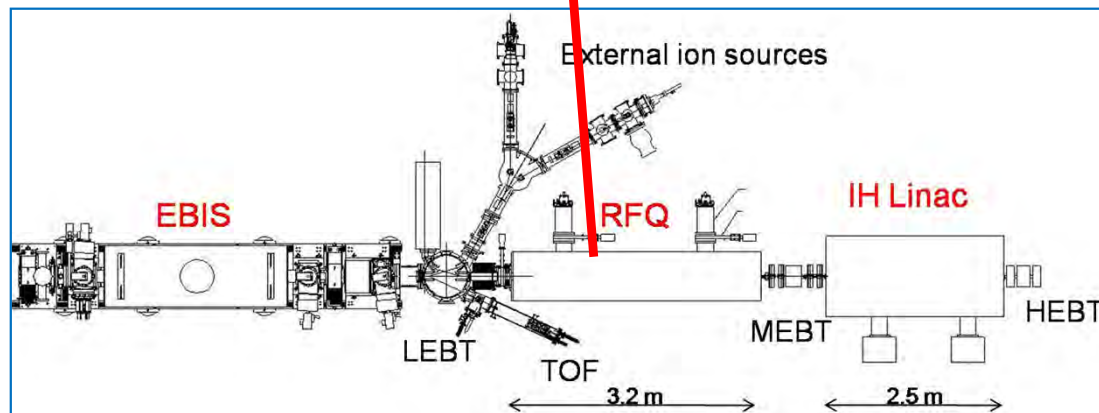
RFQ from IAP, Frankfurt (A. Schempp)



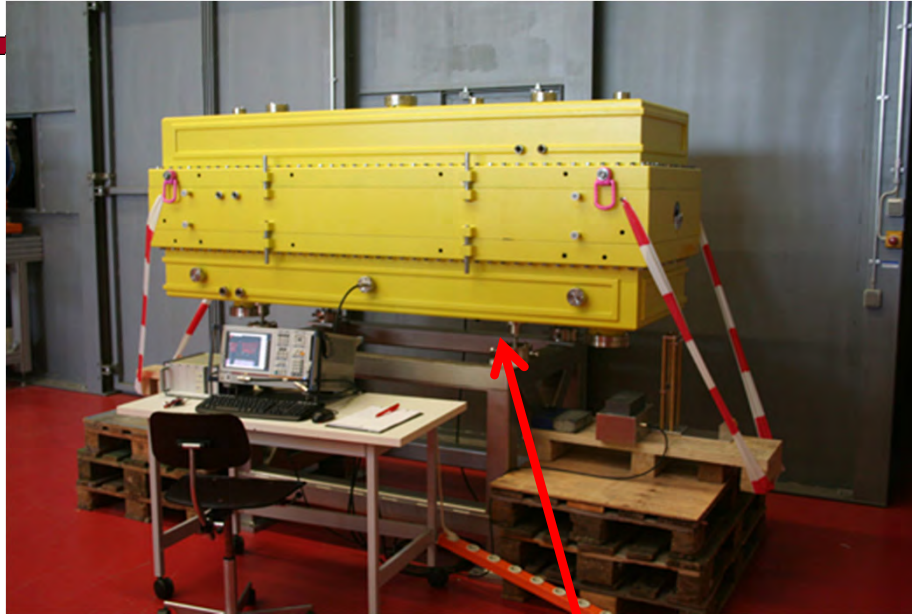
100 MHz

Accelerates the beam from 17 keV/u to 300 keV/u

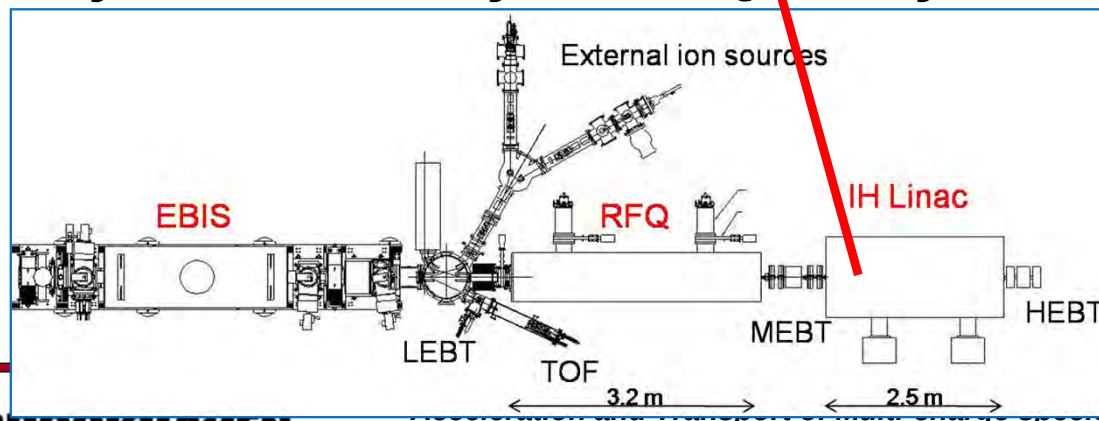
Fabricated by NTG



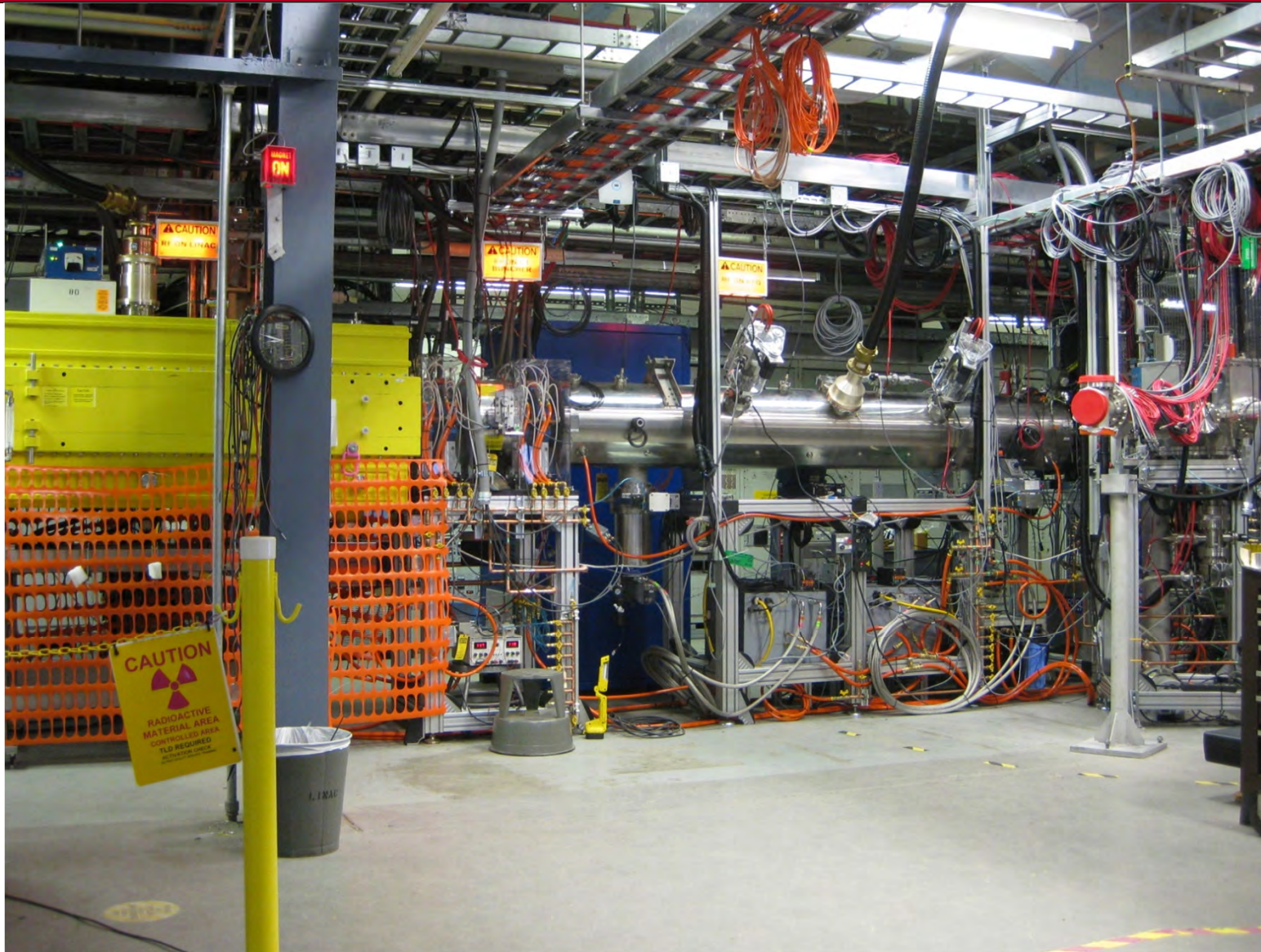
Linac from IAP, Frankfurt (U. Ratzinger)



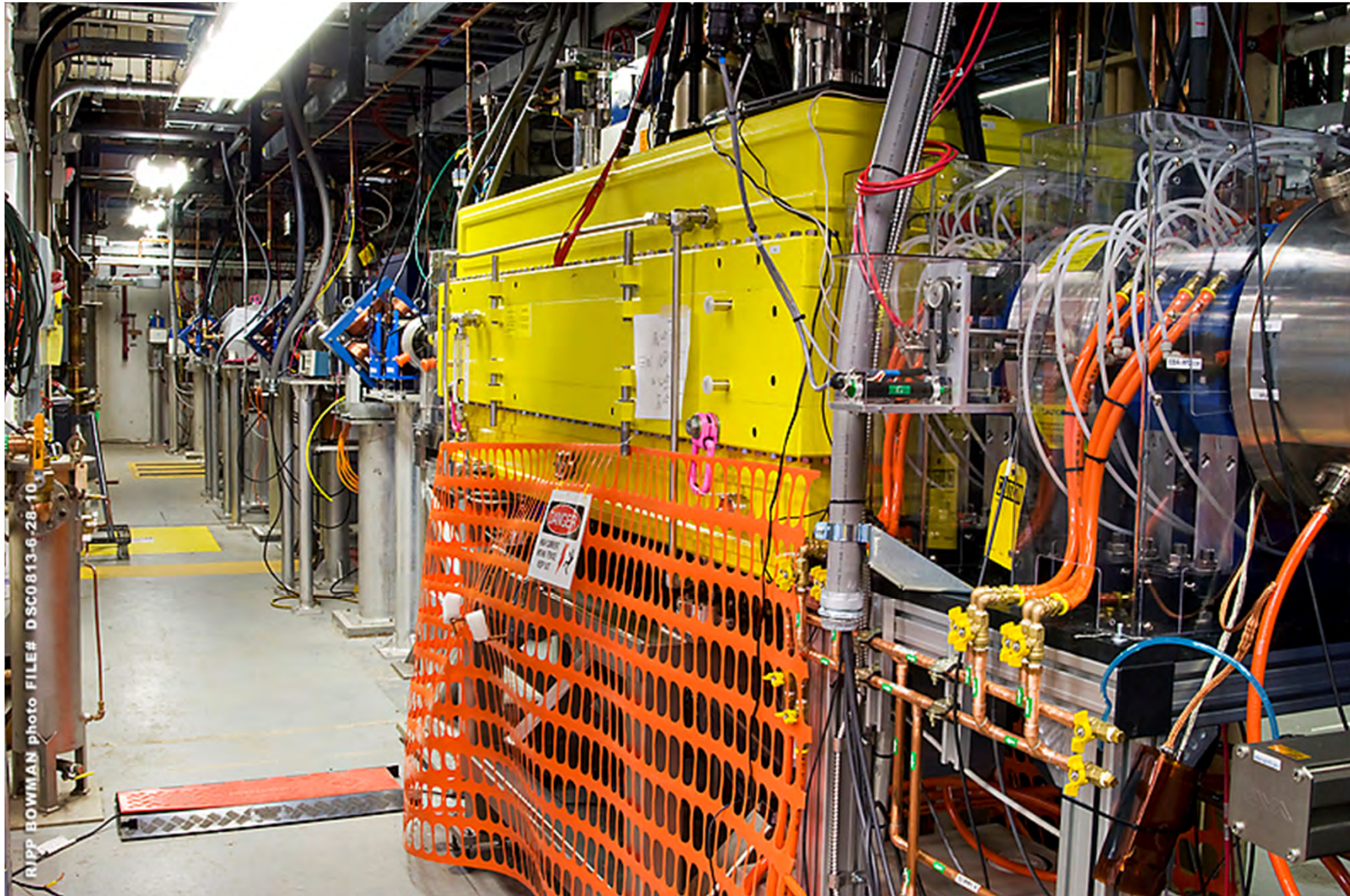
100 MHz
Accelerates the beam from 300 keV/u
to 2 MeV/u
Cavity Fabricated by PINK; Quad by Bruker



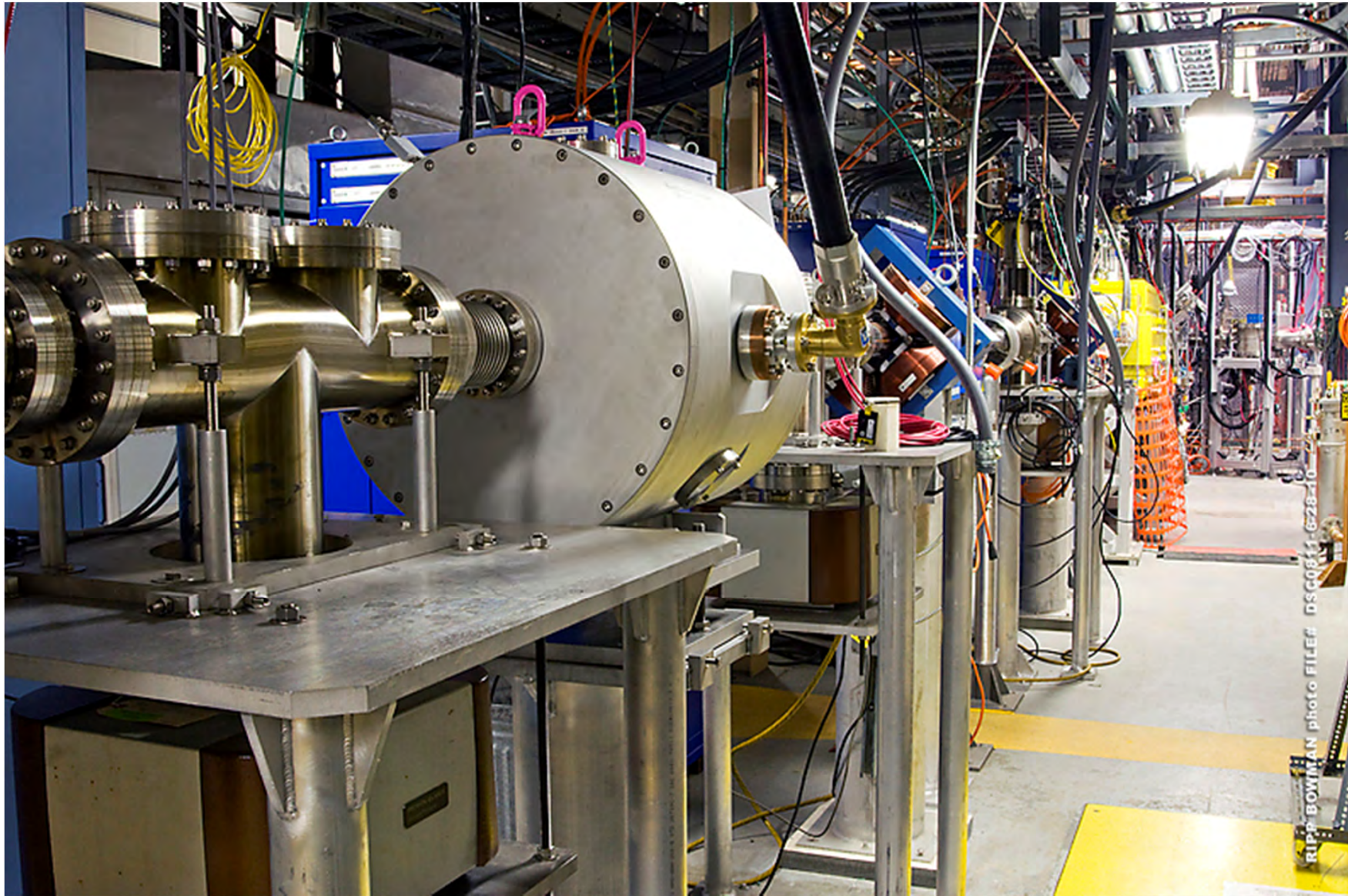
RFQ. MEBT, and Linac



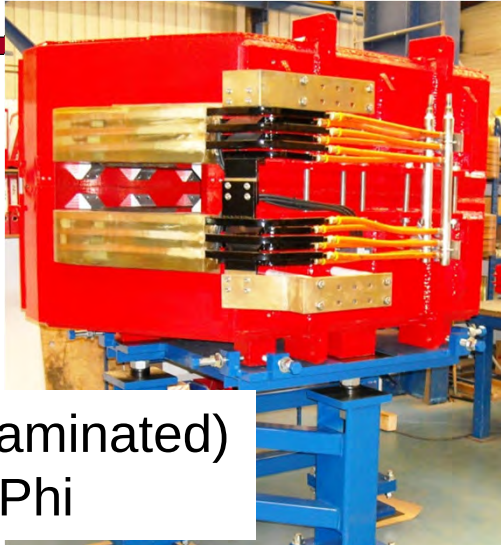
Linac and EBIS-to-Booster transport



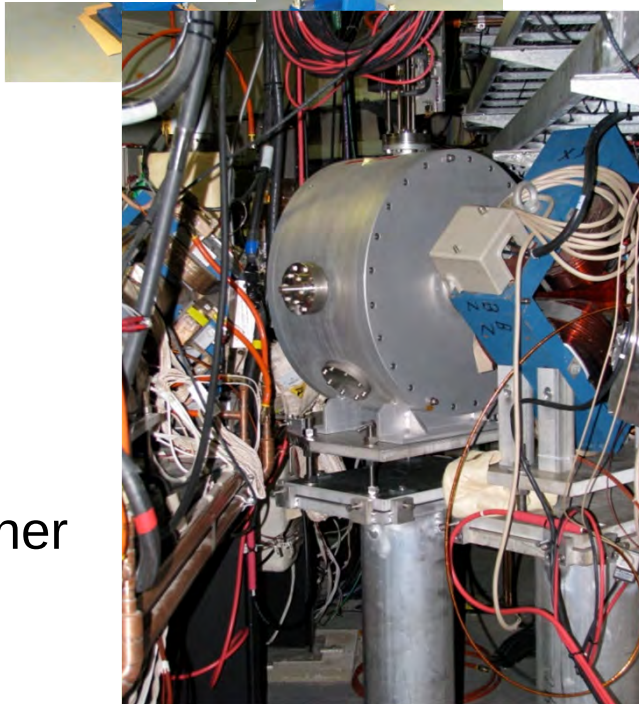
Buncher C-2 in transport line



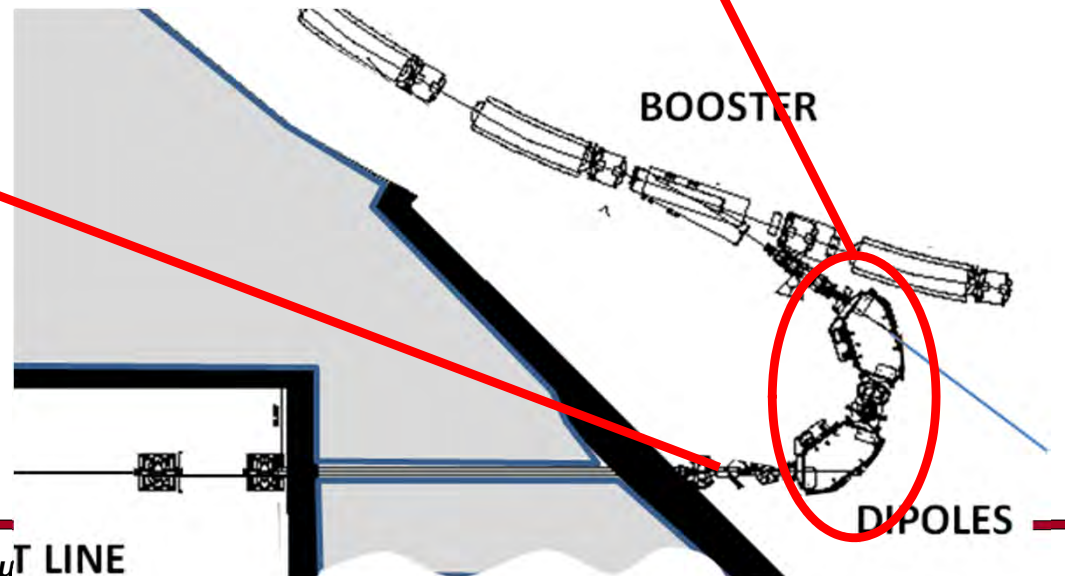
Beamline in Booster



Dipole (laminated)
- SigmaPhi



Buncher

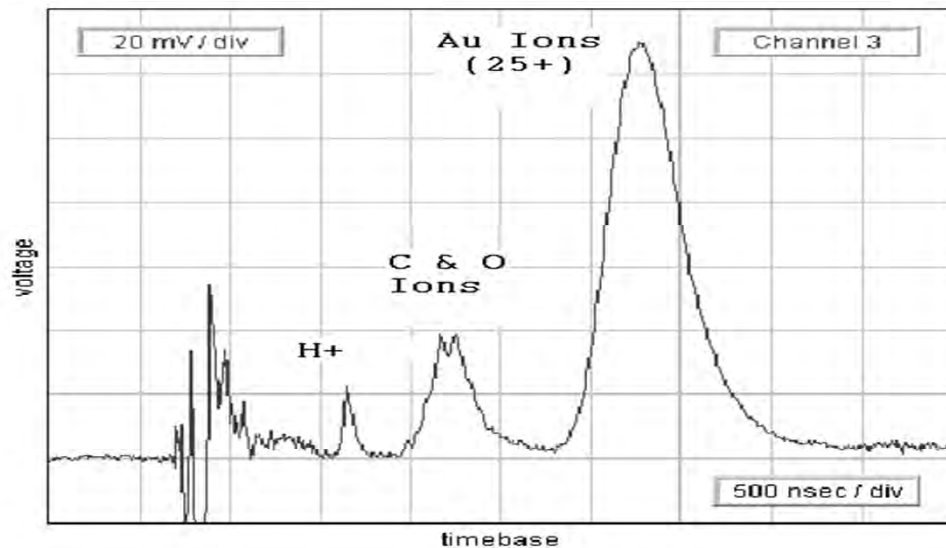


Ion Beam Delivered to NSRL and RHIC

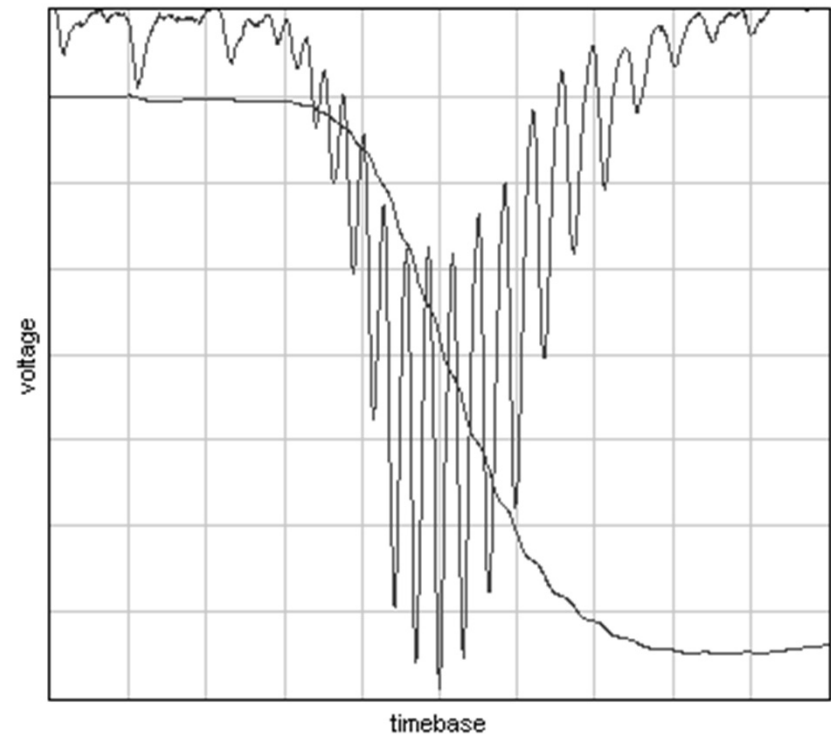


IONS	Q/m	delivered to	
${}^3\text{He}^2$	0.6667	AGS	
${}^4\text{He}^2$	0.5000	NSRL	
${}^{12}\text{C}^6$	0.5000	NSRL	
${}^{16}\text{O}^8$	0.5000	NSRL	
${}^4\text{He}^1$	0.2500	NSRL	
${}^{20}\text{Ne}^{+5}$	0.2500	NSRL	
${}^{40}\text{AR}^{+10}$	0.2500	NSRL	
${}^{63}\text{Cu}^{11}$	0.1746	RHIC	4.8e9
${}^{40}\text{AR}^{+7}$	0.1750	AGS	
${}^{48}\text{TI}^{+9}$	0.1875	NSRL	
${}^{56}\text{Fe}^{+10}$	0.1786	NSRL	
${}^{84}\text{Kr}^{+20}$	0.2439	NSRL	
${}^{131}\text{Xe}^{+30}$	0.2290	NSRL	
${}^{181}\text{Ta}^{+40}$	0.2210	NSRL	
${}^{197}\text{Au}^{+32}$	0.1624	RHIC, NSRL	1.6e9
${}^{238}\text{U}^{+39}$	0.1638	RHIC	1.3e9

Why Multiple Ion Species

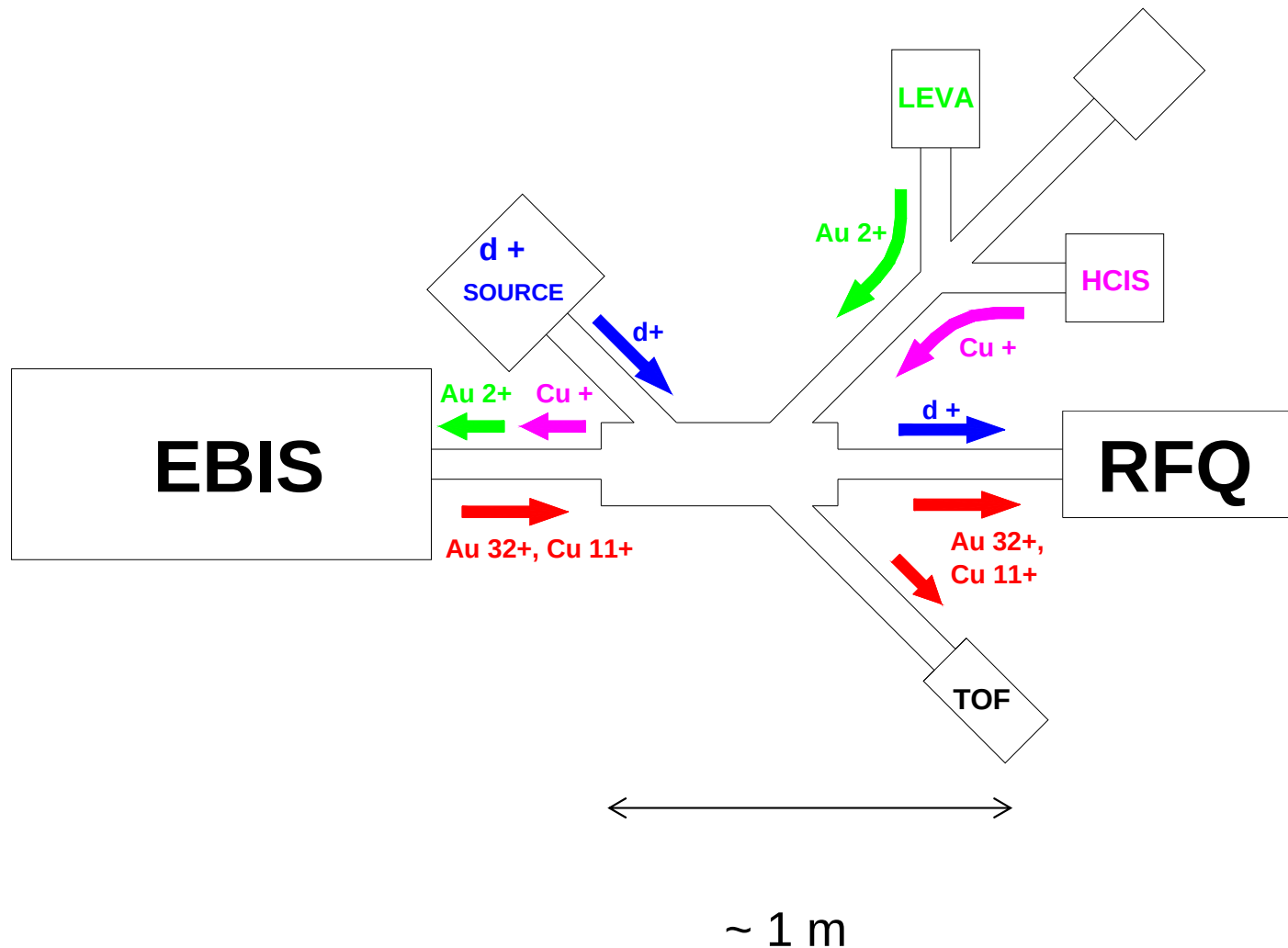


- Full ion beam sampled and collected on Faraday Cup
- $I_e = 7\text{A}$;
- 10 ms confinement
- Au = 83%; C&O = 15%; H = 2%



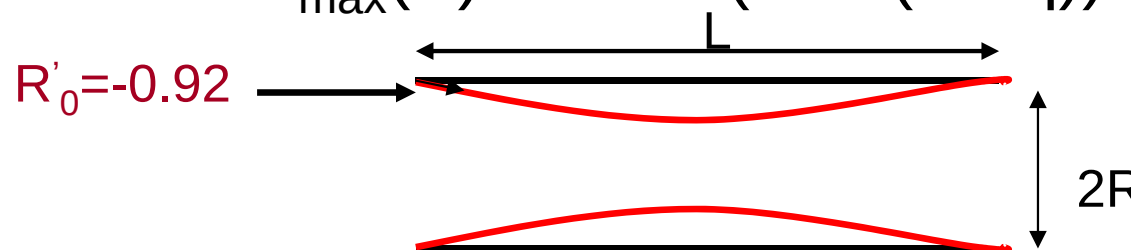
Au time-of-flight charge state distribution; Peak at Au 31+
 $I(e)=7.6\text{ A}$; 65ms confinement

Drift space needed for injection in LEBT



LEBT Length

- Distance needed between two lenses for external ion injection (L) ~ 1 m.
- Beam radius (R) ~ 2.0 cm, Lenses (solenoid) aperture radius 5.0 cm
- Maximum beam current for SC dominated drift (L), maximum beam radius (R) and initial slop $R'_0 = -0.92$ given by

$$I_{\max}(\text{A}) = 1.166 * (mc^2 / (30 * q)) * \beta^3 \gamma^3 (R/L)^2$$


= 20 mA for Au^{+32}

= 6 mA for D

= 5 mA for ${}^3\text{He}^{+2}$

Alternative LEBT: Charge Separation before RFQ

- Dispersion 0.8 meter
- Sufficient for charge separation
- Magnets has to float to 40 kV
- Unnecessary complication
- Cost

90° bend with gradient ($n=0.50$)
 $\rho=300$ mm, edge angle= 4°
 Gap 100 mm

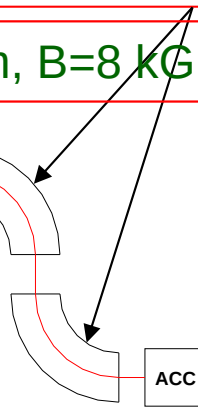
$L=241$ mm, $B=8$ kG

EBIS →

SOL

ACC

$V=-15$ kV
 $L=150$ mm

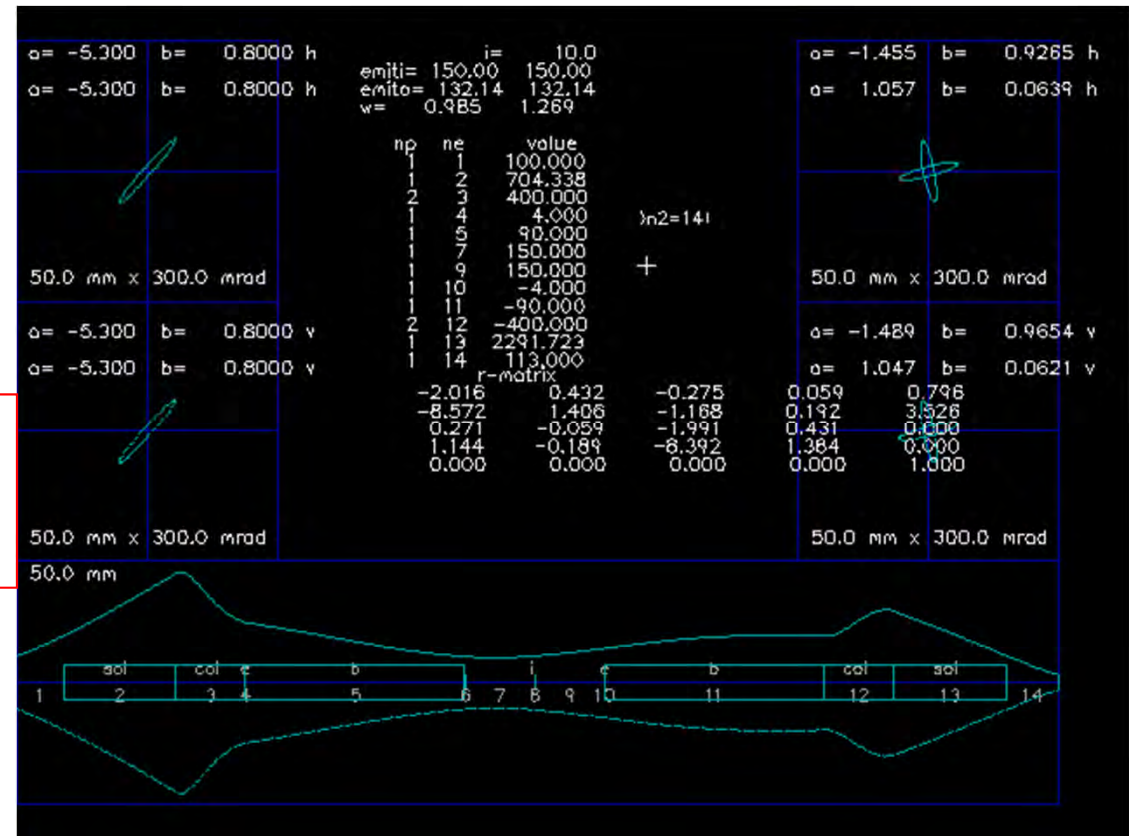


ACC

SOL

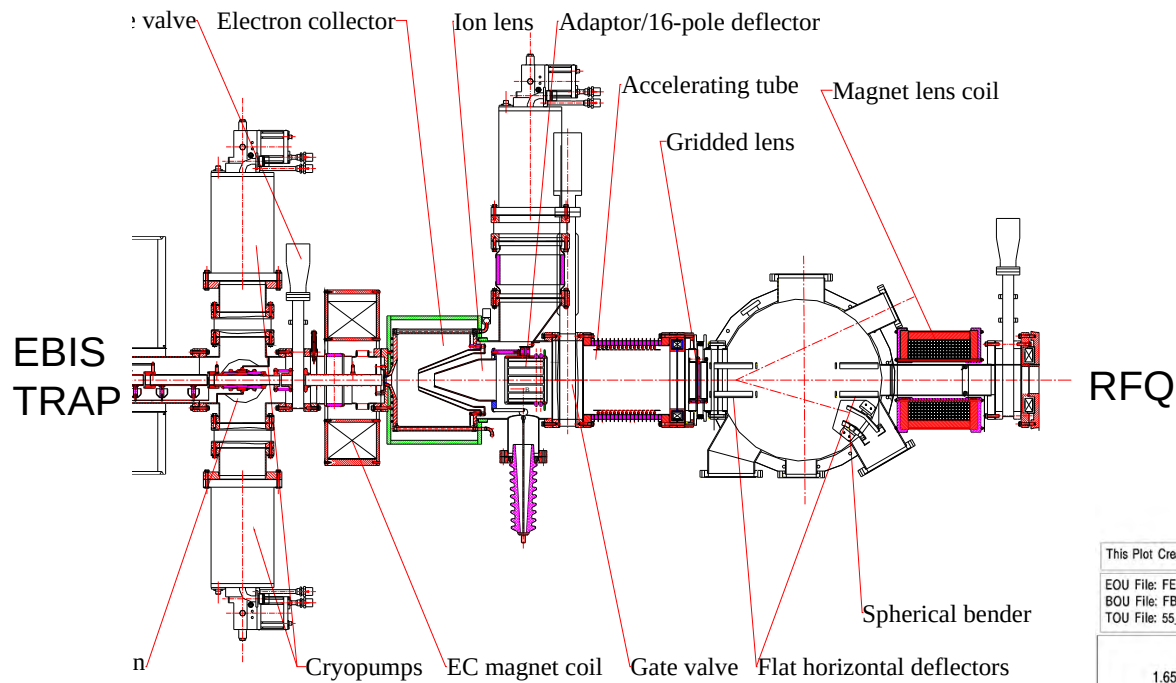
RFQ

$V=15$ kV, $L=150$ mm



Two 90 degree gradient magnet with 4° edge angle, 10 cm gap, Float at 40 kV for ${}^3\text{He}^{+2}$

Matching to the RFQ



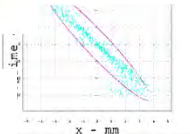
This Plot Created On: March 24, 2006

Optics parameters:

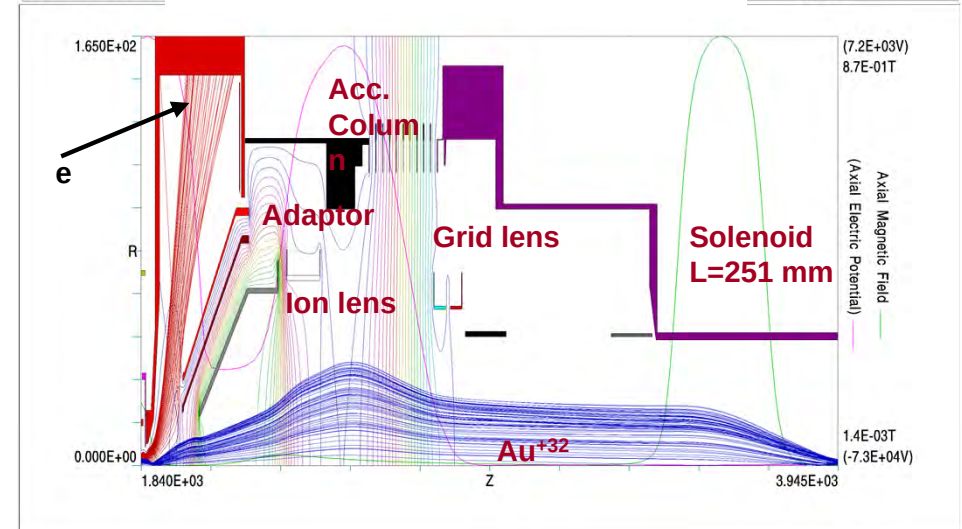
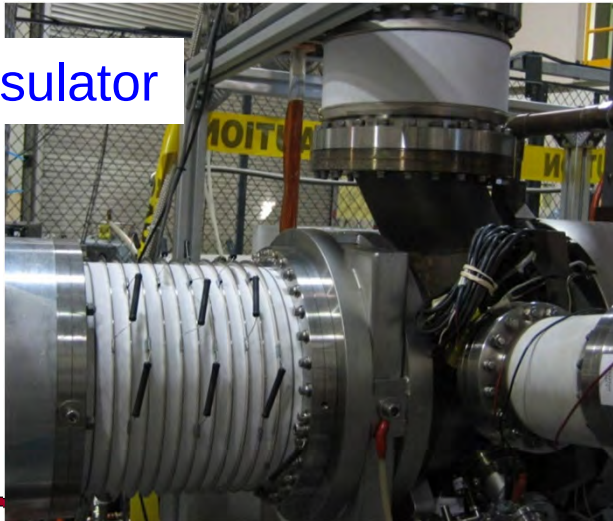
$E_{RMS\_norm\_init} = 0.068 \text{ pi.mm.mrad}$
 $E_{RMS\_norm\_final} = 0.0803 \text{ pi.mm.mrad}$

EOU File: FE55_0_67_500
BOU File: FBLEBT_6.BOI
TOU File: 55_0_67_1_78E

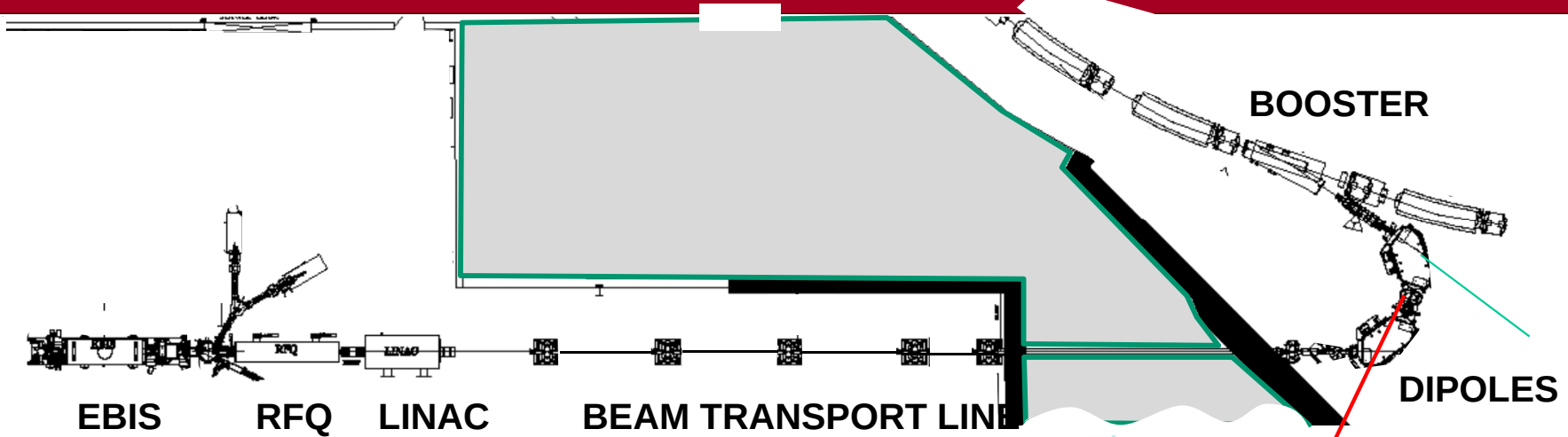
LEBT Optics, Au⁺³²



100 kV insulator



Charge Discrimination



At this location

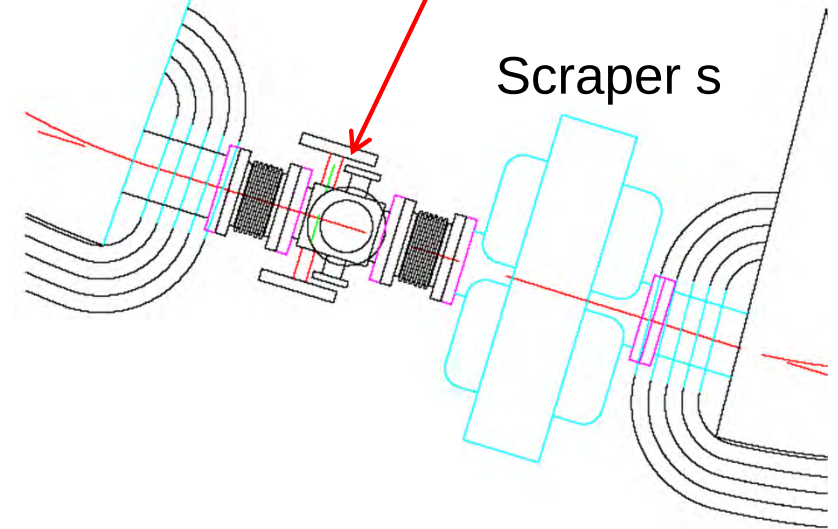
- $R_{16}=1.1$ m

- $2r_0=2.2$ mm

-Resolution = 500 @2MeV/u

310@0.3 MeV/u

-Efficiency =100%

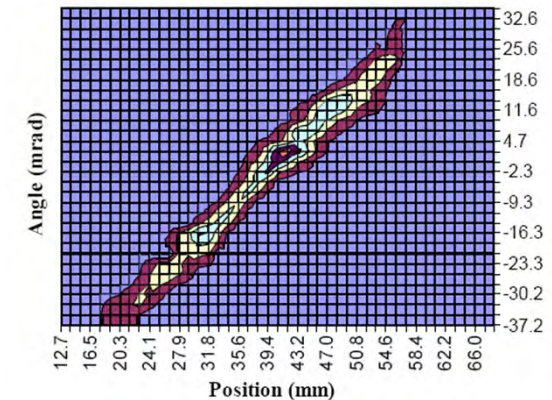


Measured emittance of a 1.7 ma Au beam

We have assumed emittance scales as Q/m for different ion Species

$$\varepsilon(n) = 0.16 * r^2 * B_z * (Q/M) \text{ m-rad (100\% of beam)}$$

=> He^{+2} emittance will ~3 times of Au^{+32}



$\varepsilon(n, \text{rms}) = 0.1 \pi \text{ mm mrad.}$

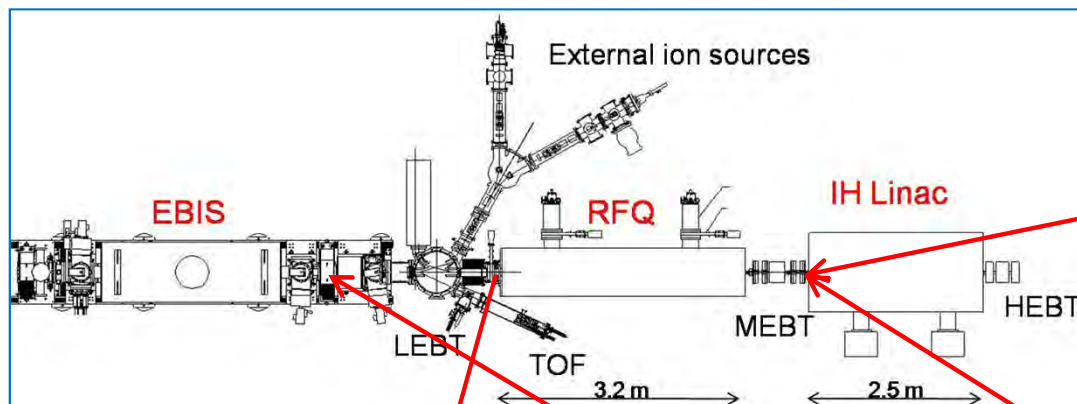
Emittance Budget for the EBIS-Injector

	Vertical $\pi \text{ mm mrad (N, 95\%)}$	Horizontal $\pi \text{ mm mrad (N, 95\%)}$
Booster*	4.5	10.0
Linac	1.4	1.4
Source	0.7	0.7

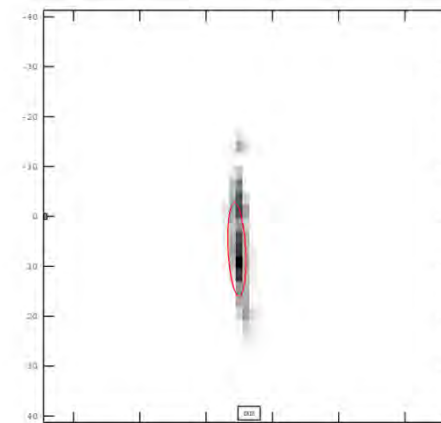
*mismatch injection

Note: Booster acceptance (VXH) $4.5 \times 15 \pi \text{ mm mrad (N,95\%)}$

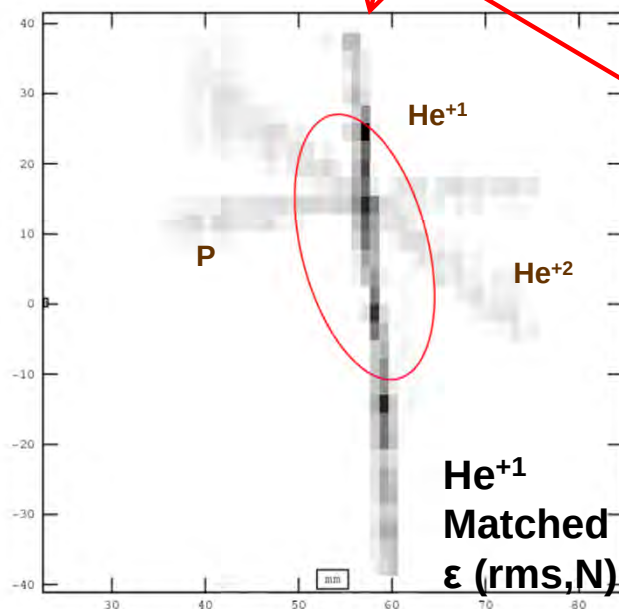
The LEBT and MEBT beam lines were each first commissioned with temporary diagnostics, before moving the RFQ, and then the linac, into place.



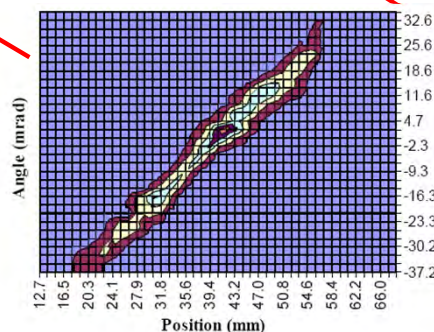
MEBT



Helium: matched condition into LINAC
 $\epsilon(n, rms) 0.13 \pi \text{ mm mrad}$

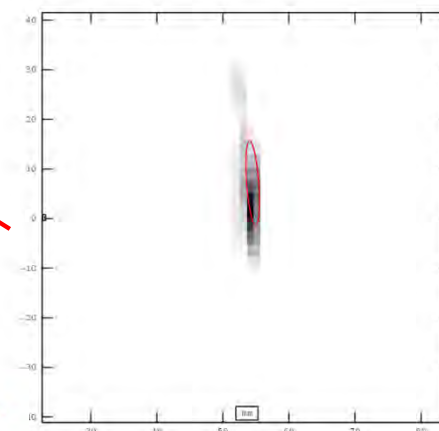


LEBT



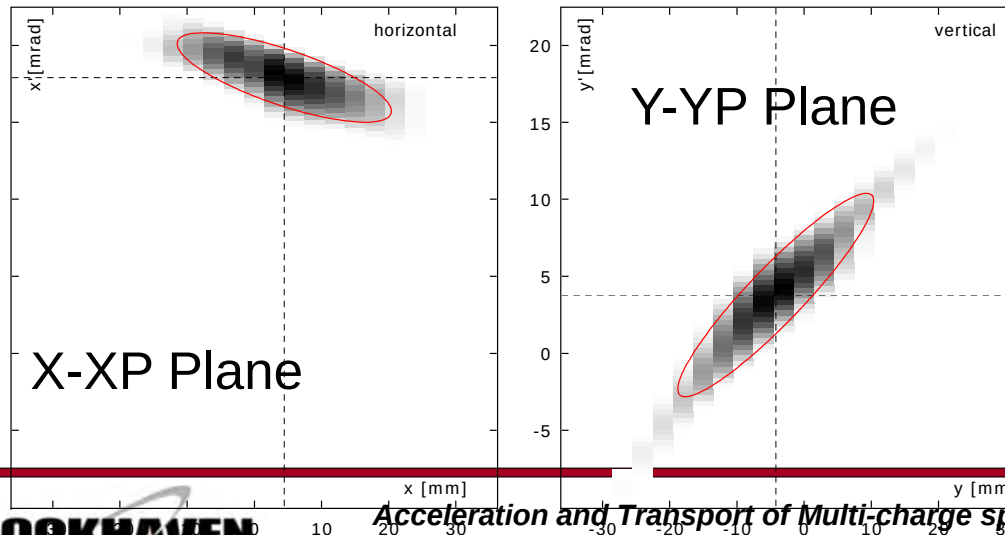
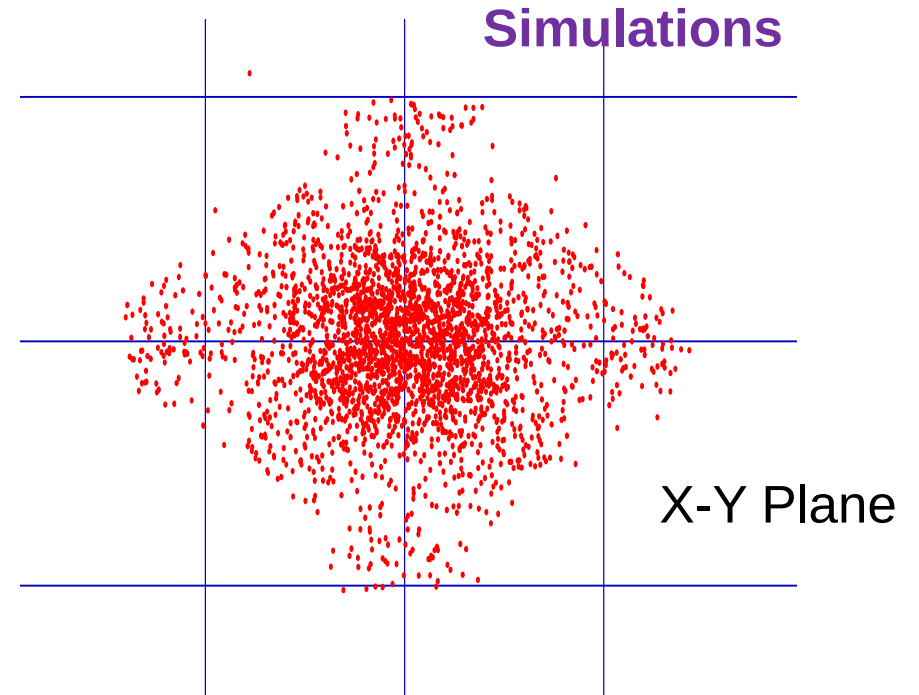
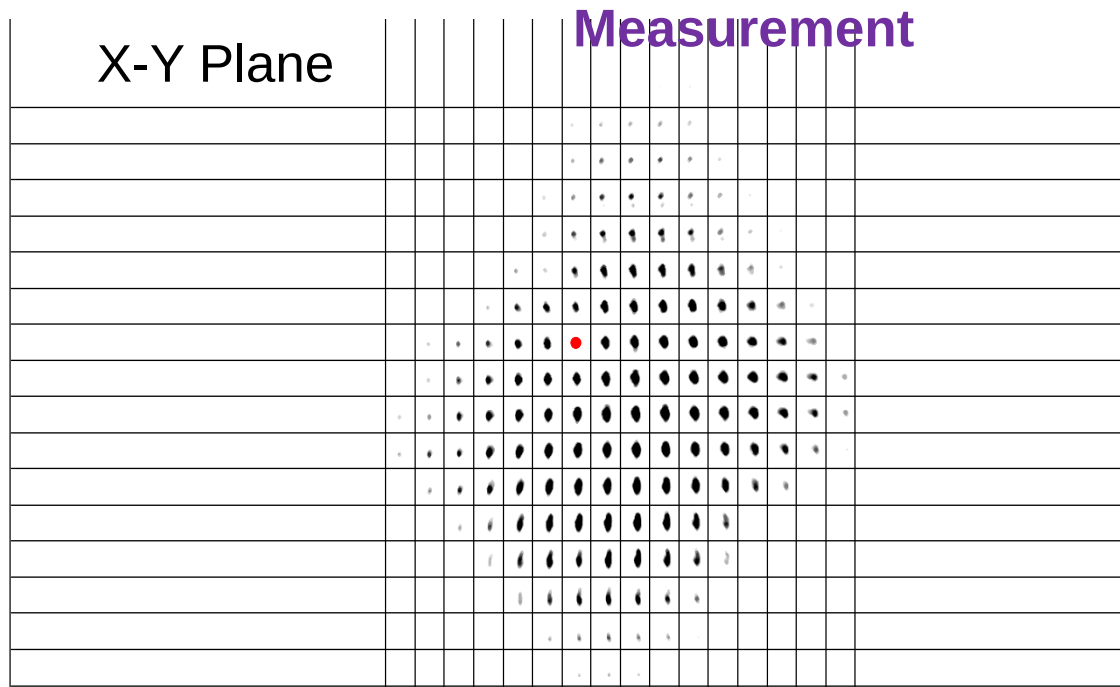
Gold: $\epsilon(n, rms) = 0.1 \pi \text{ mm mrad}$.

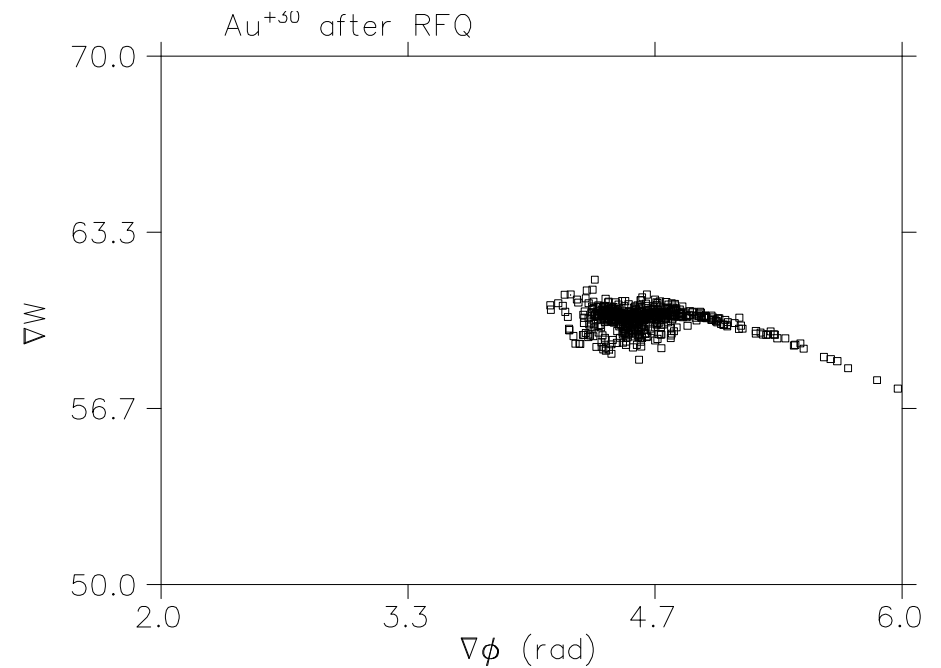
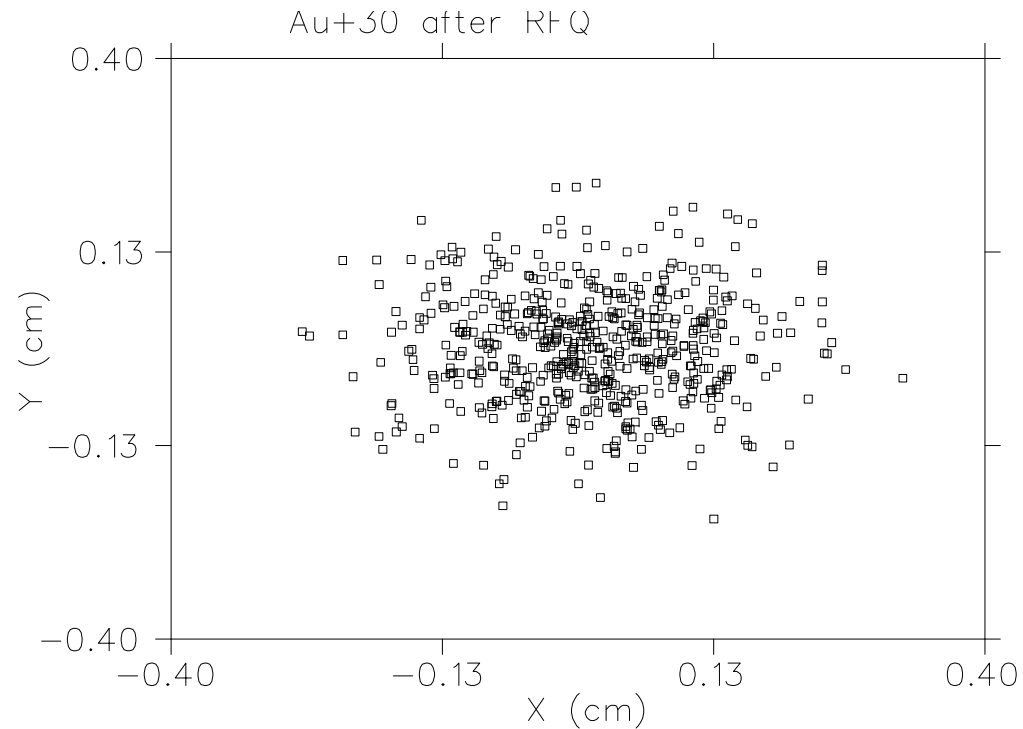
He⁺¹
Matched condition into the RFQ
 $\epsilon(rms, N) = 0.09 \pi \text{ mm mrad}$



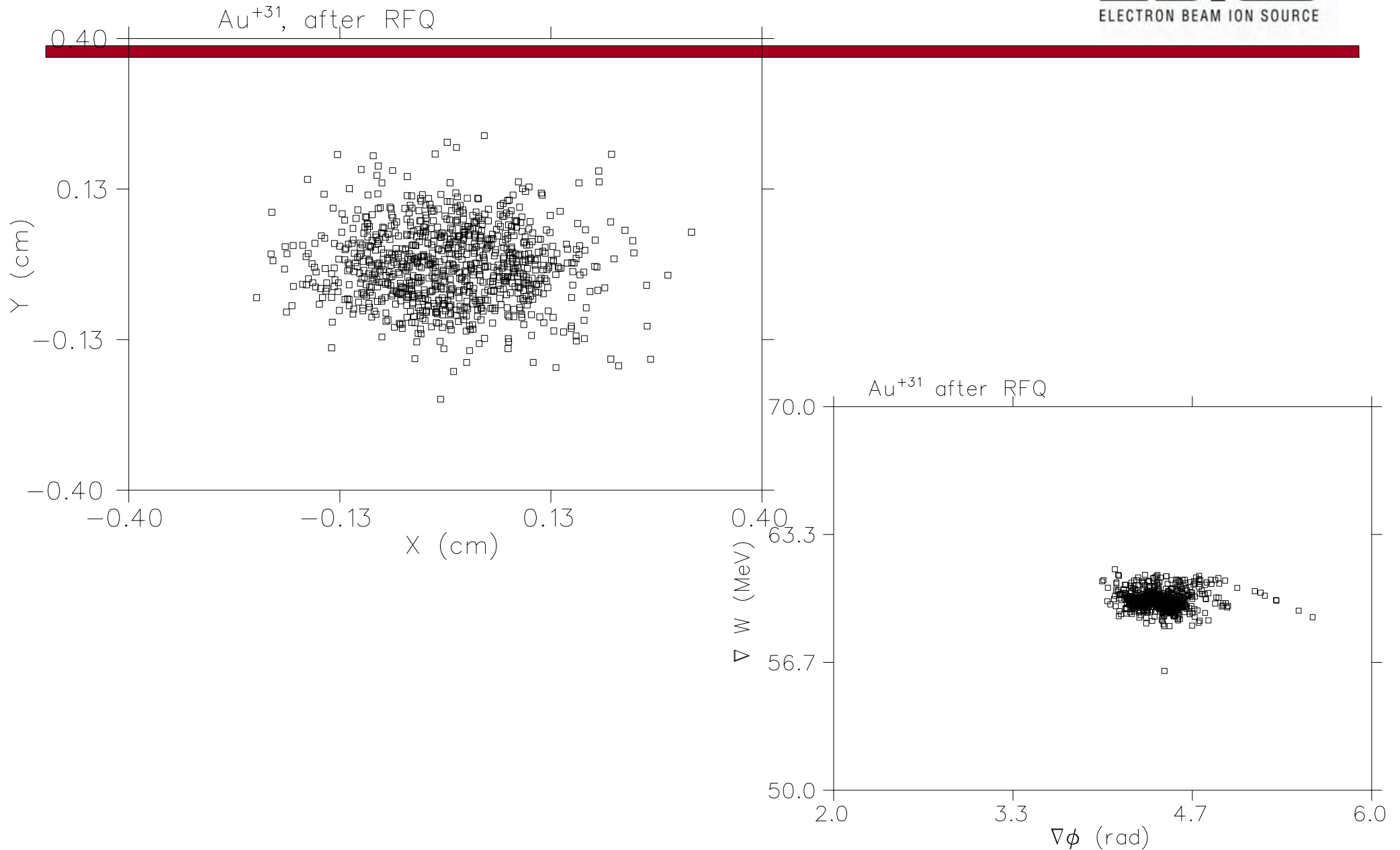
Gold: Matched conditions into linac
 $\epsilon(n, rms) 0.19 \pi \text{ mm mrad}$

Image of the Beam (U) after linac

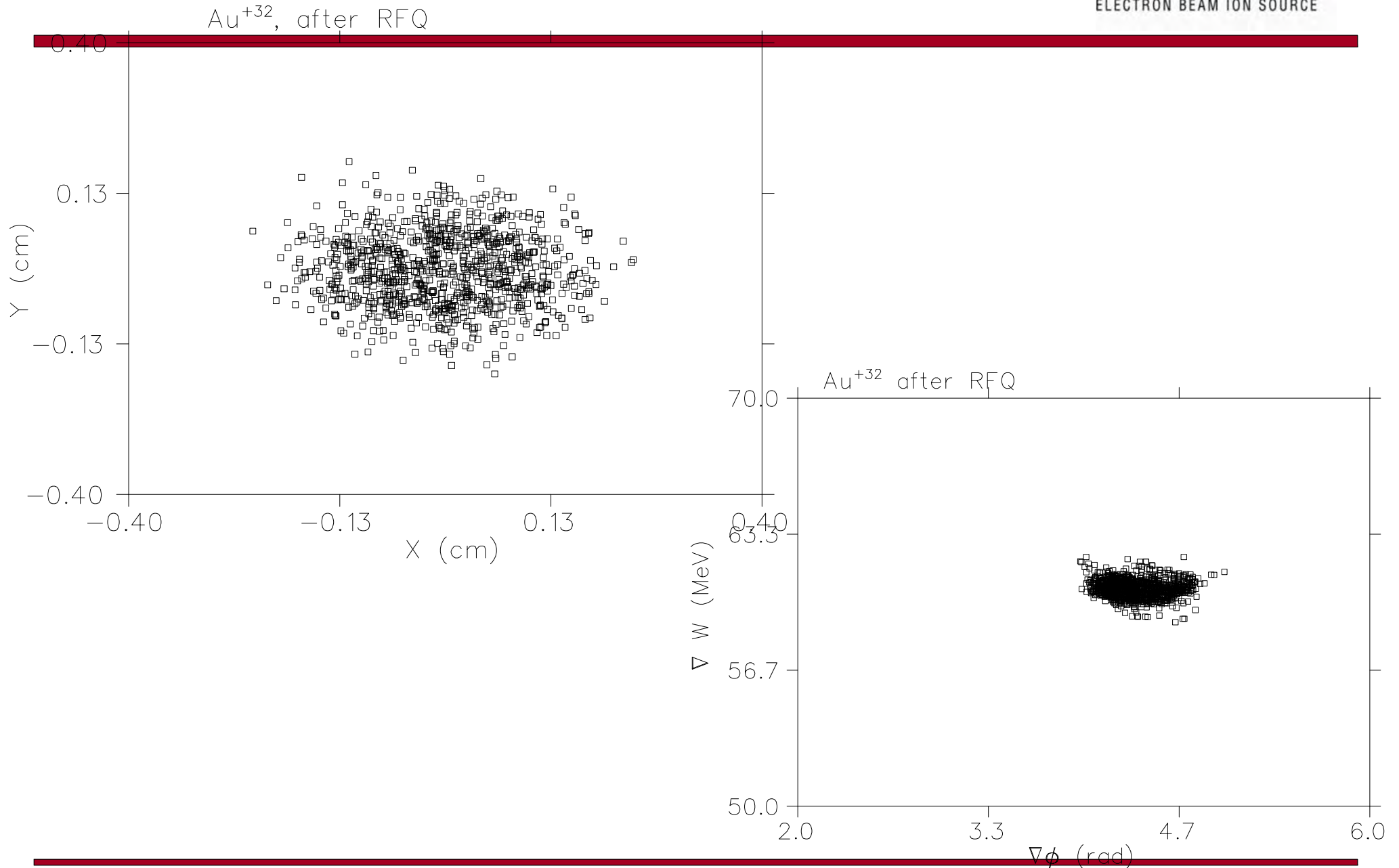


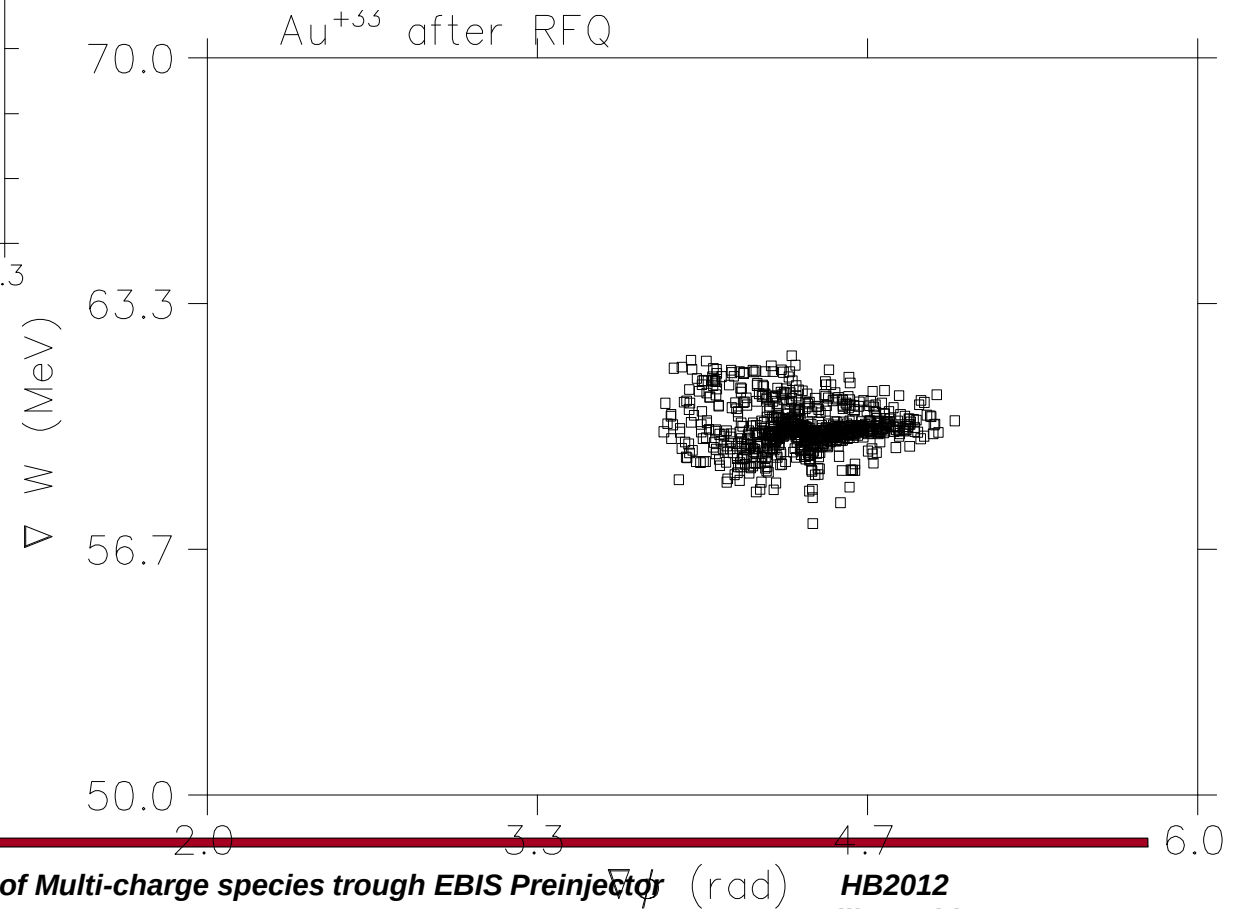
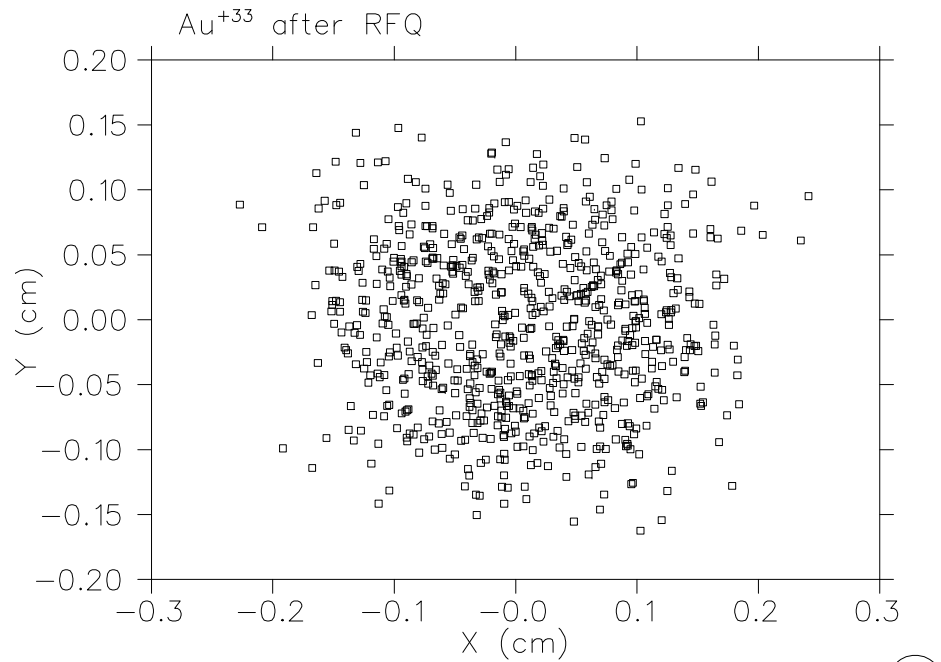


Au 31

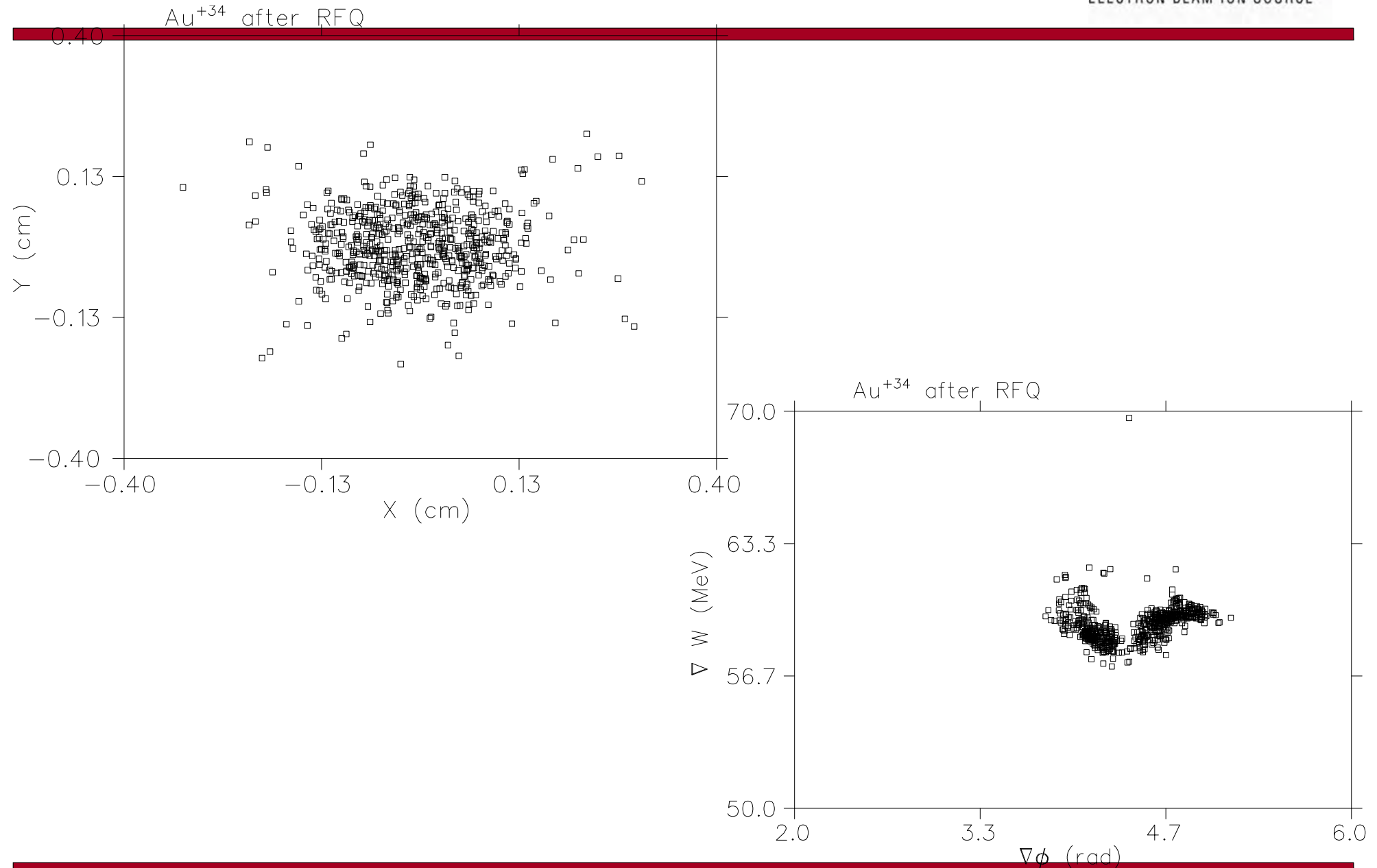


Au+32

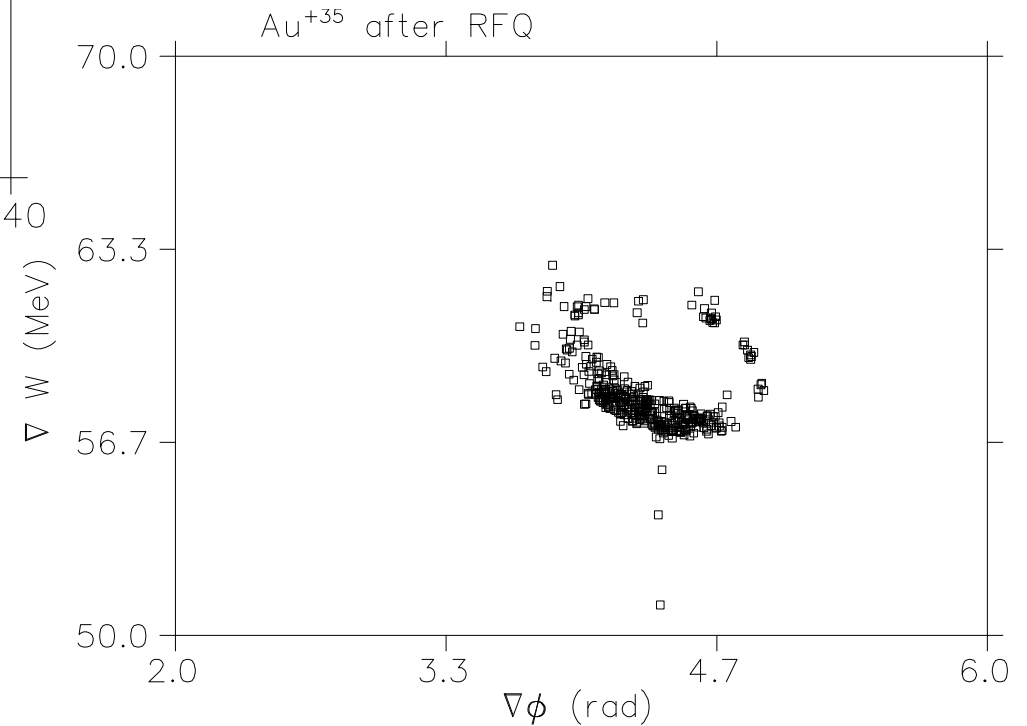
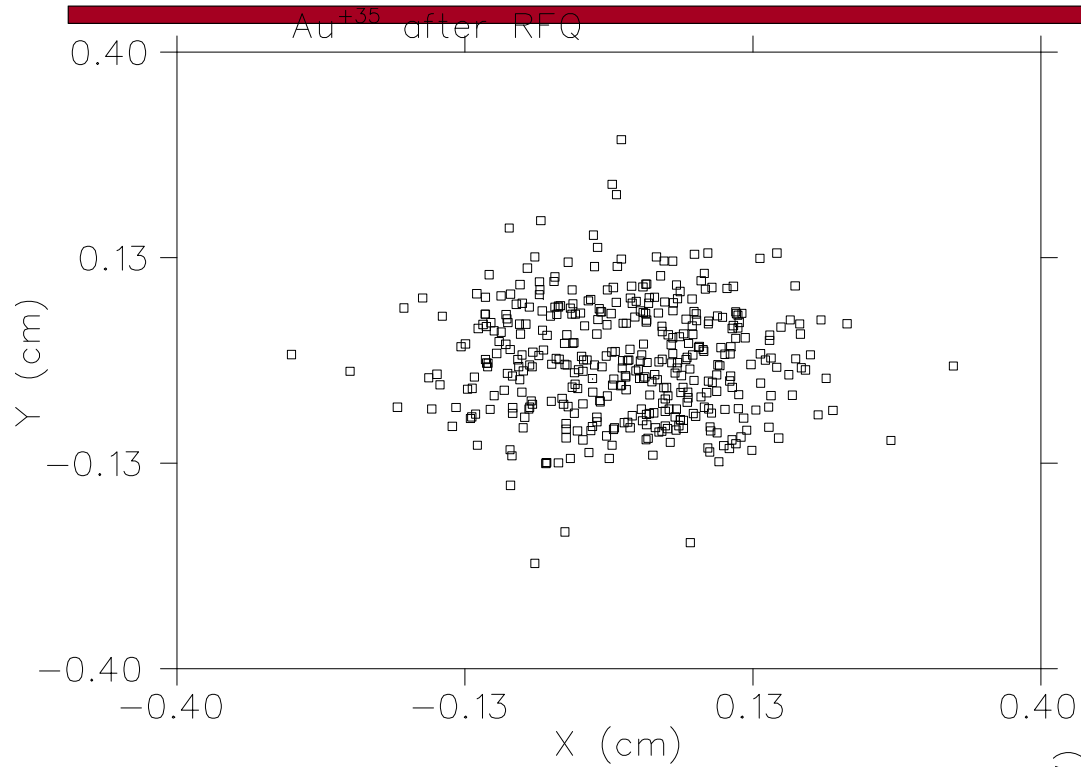




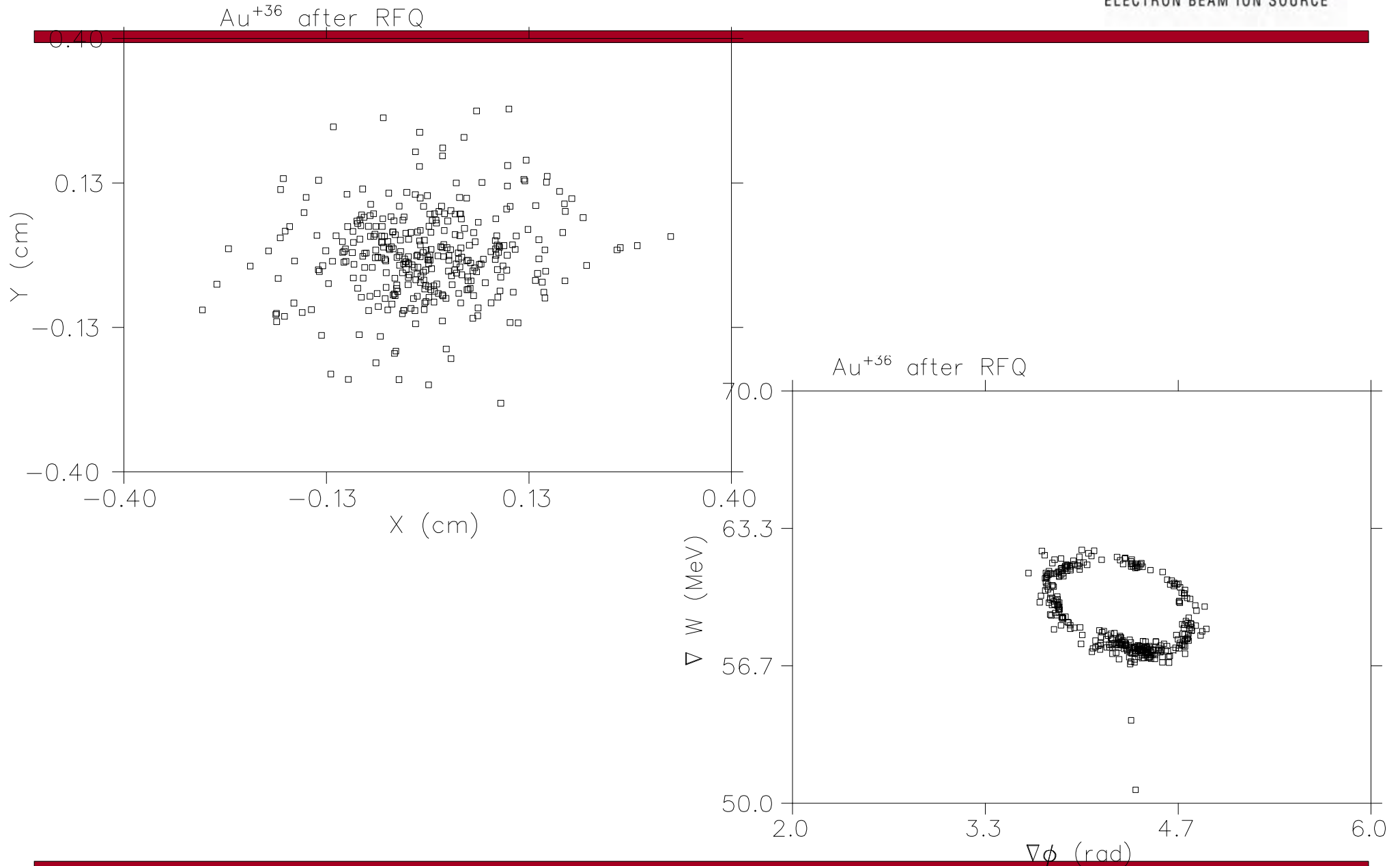
Au +34



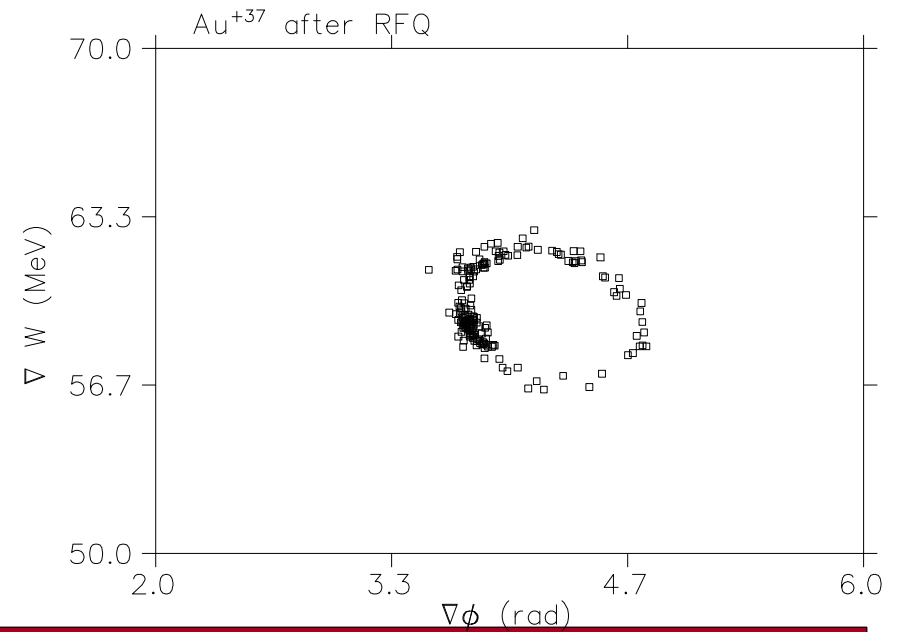
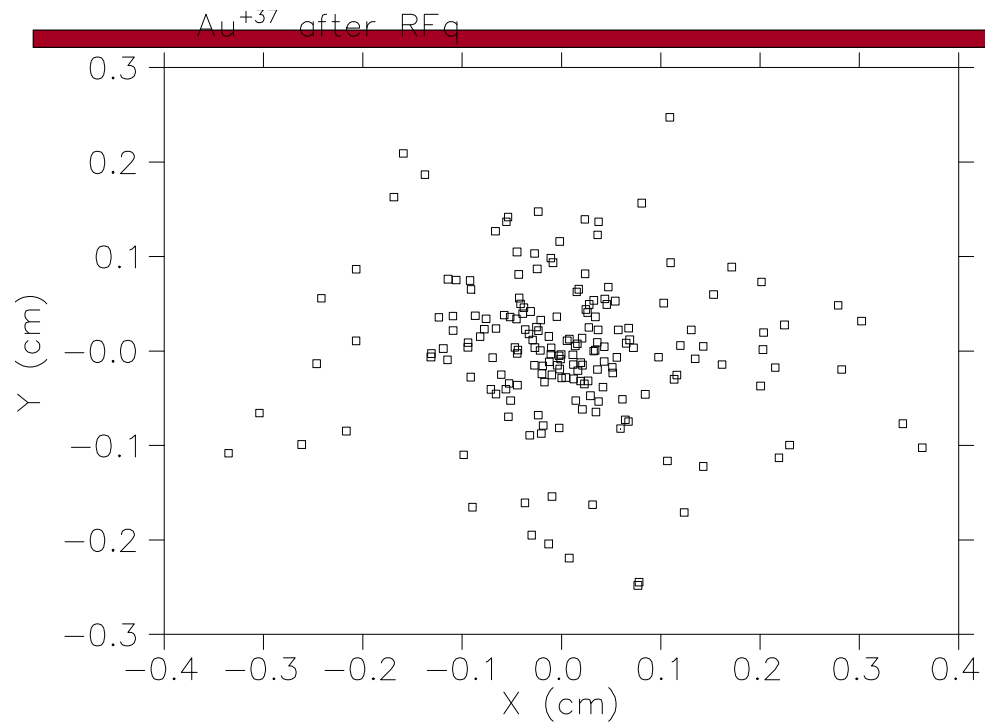
Au +35



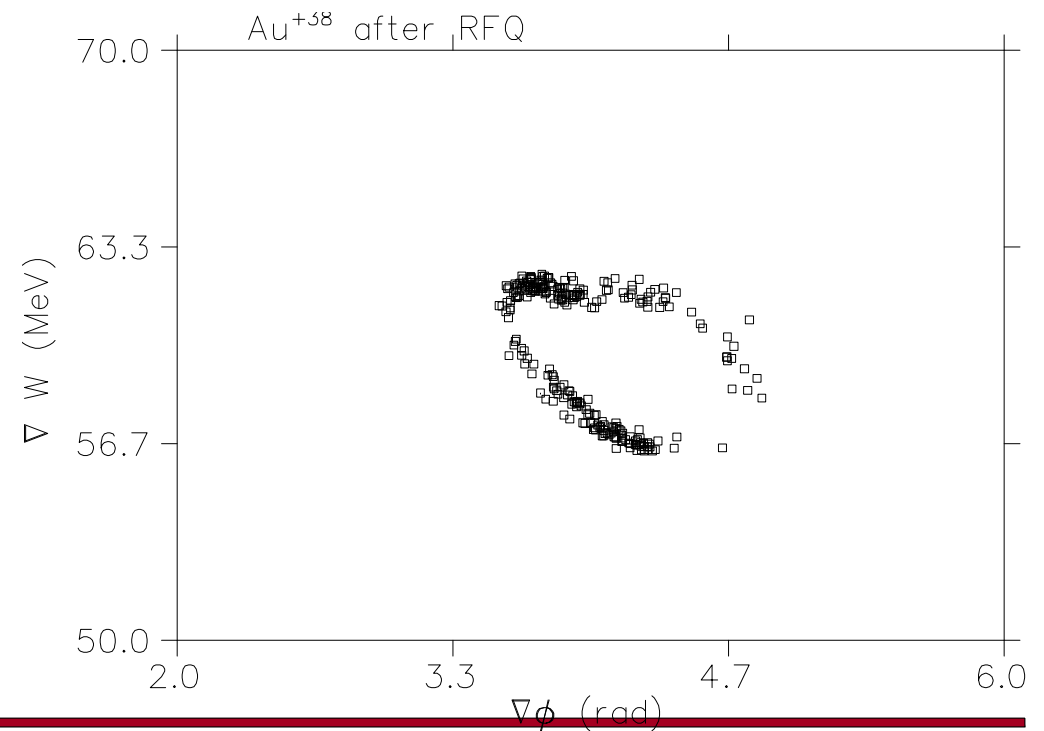
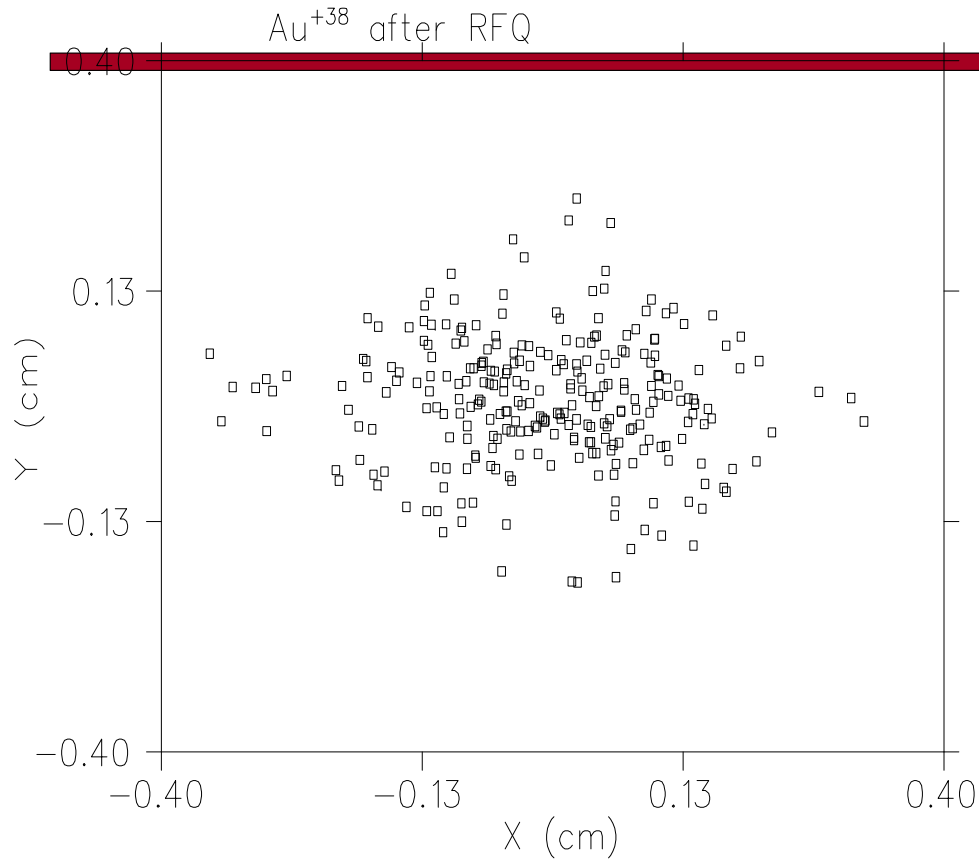
Au +36

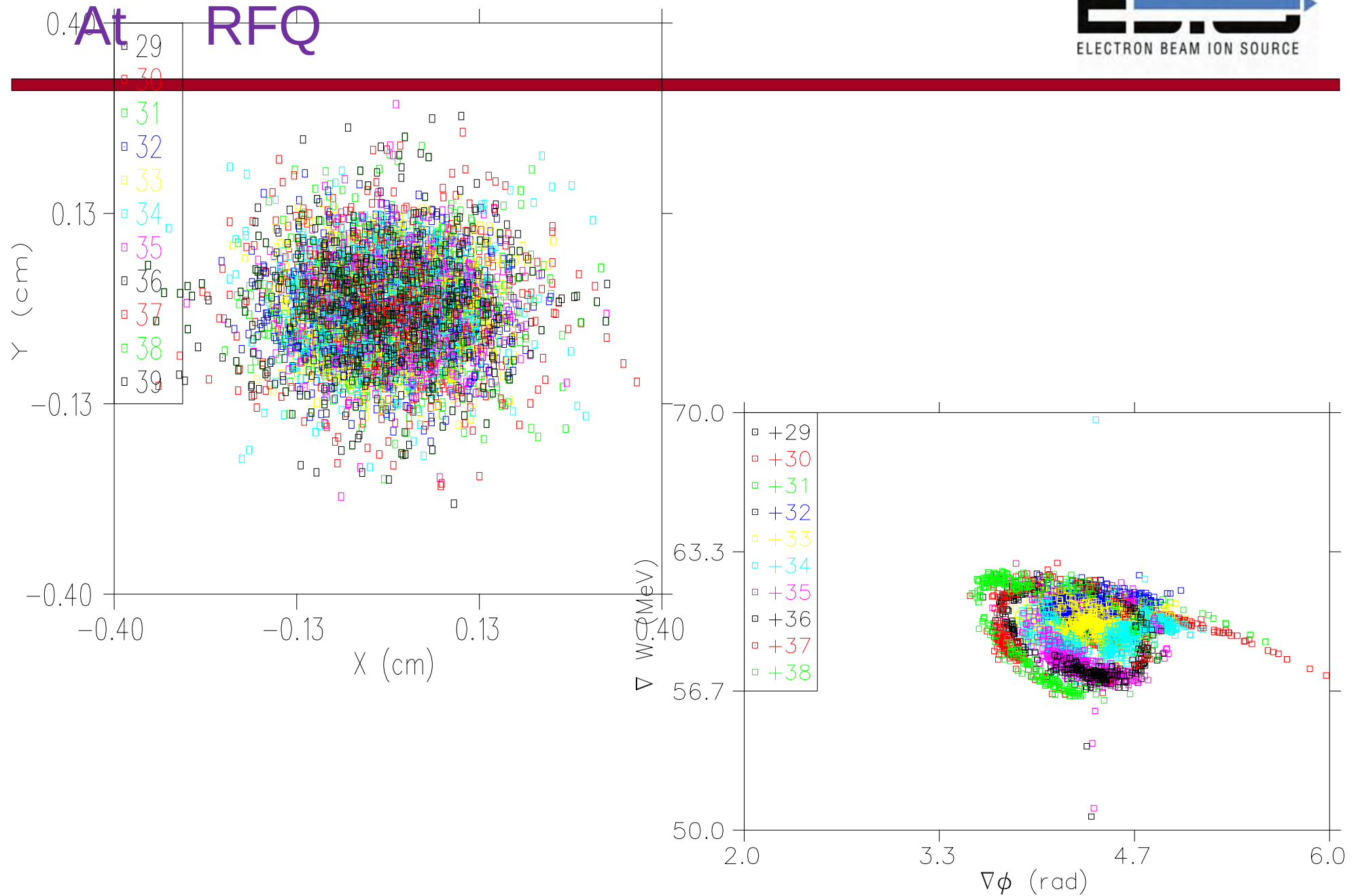


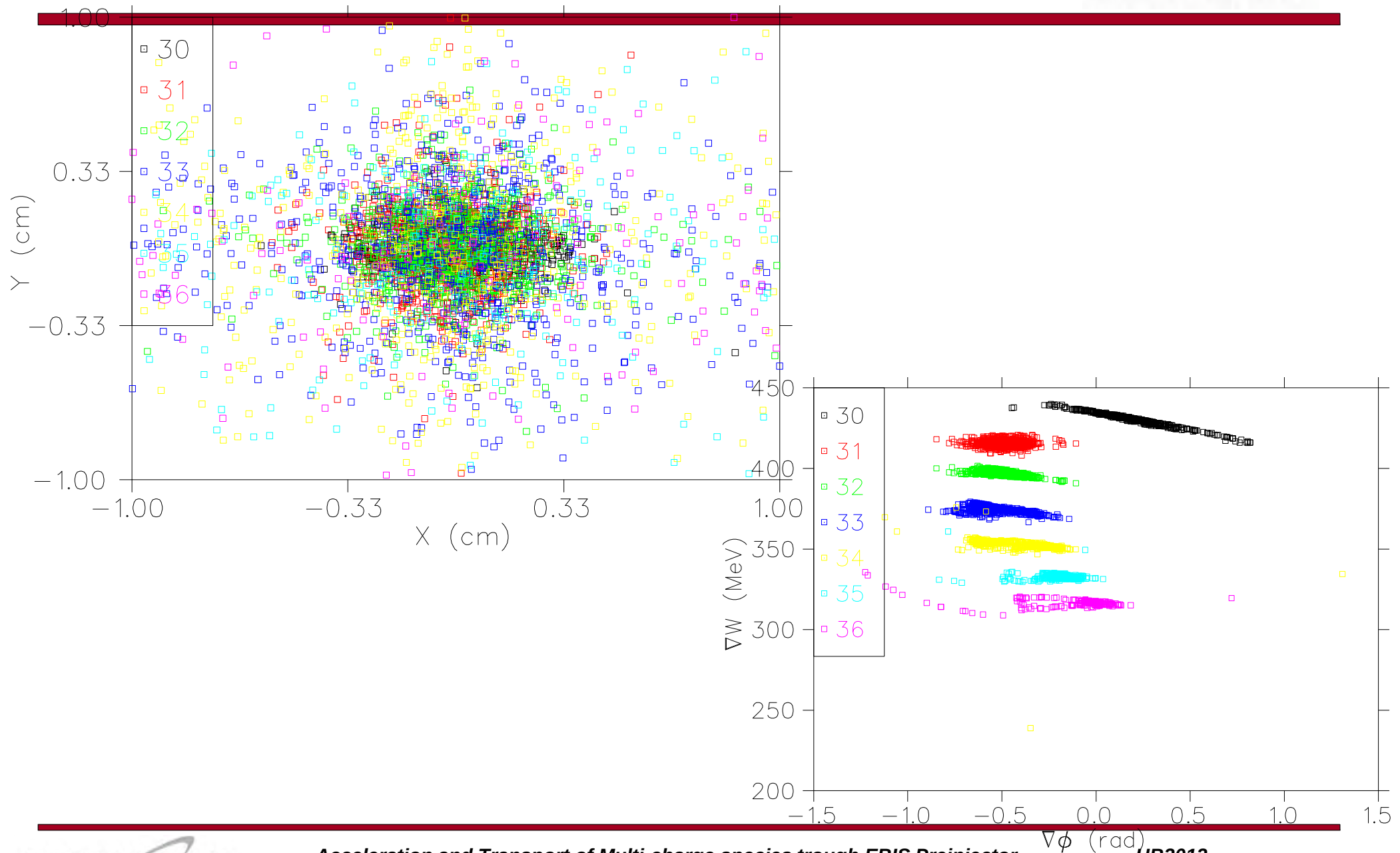
Au +37



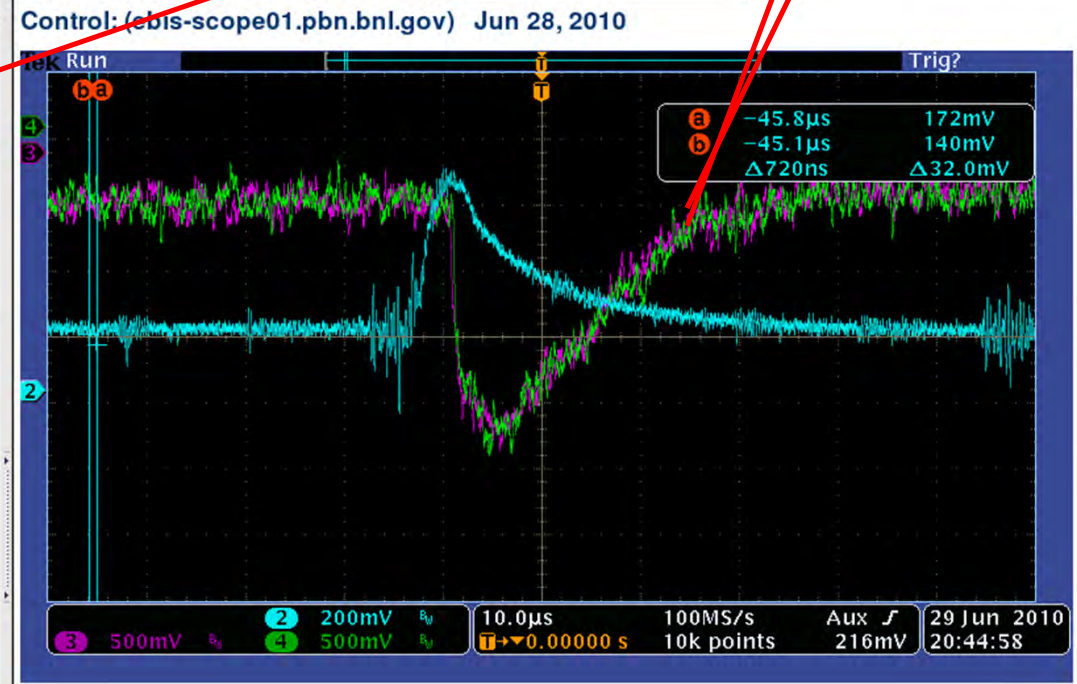
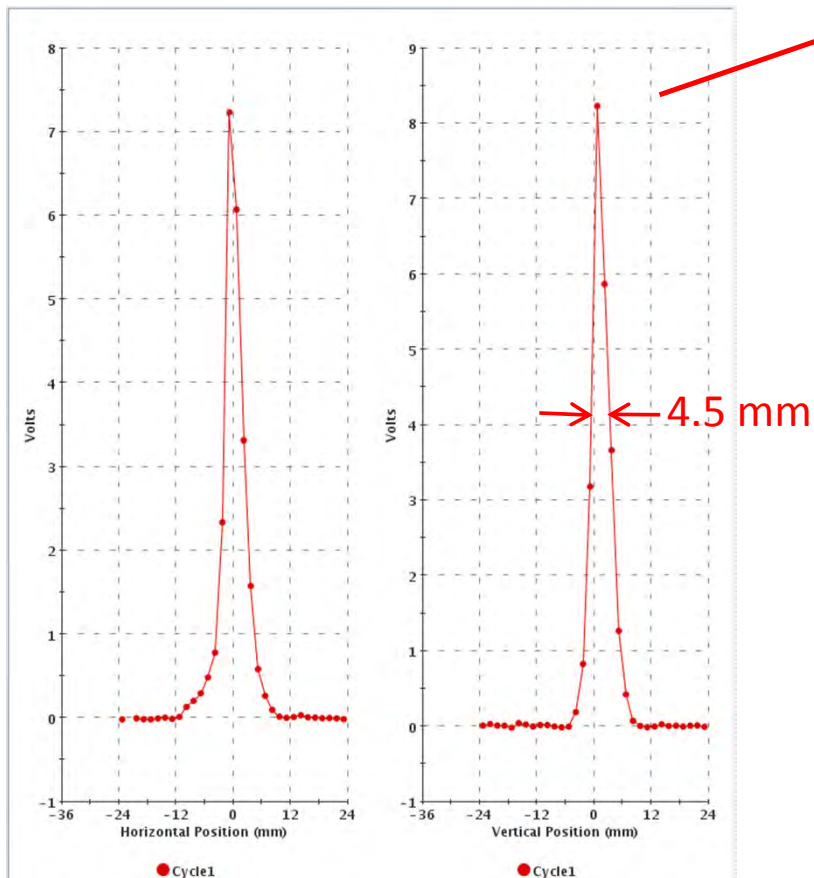
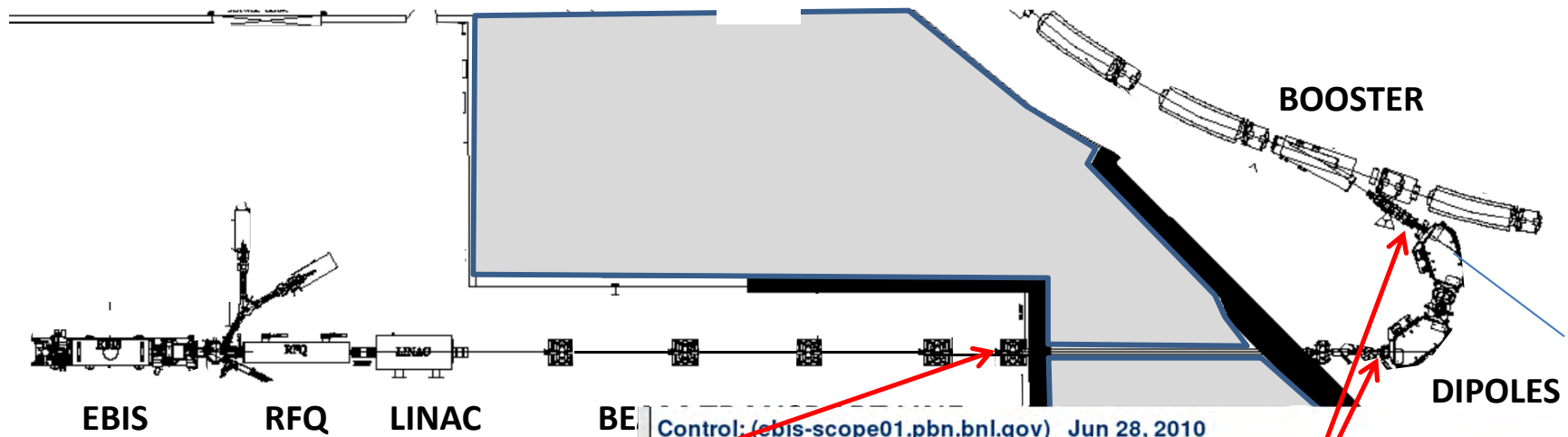
Au +38





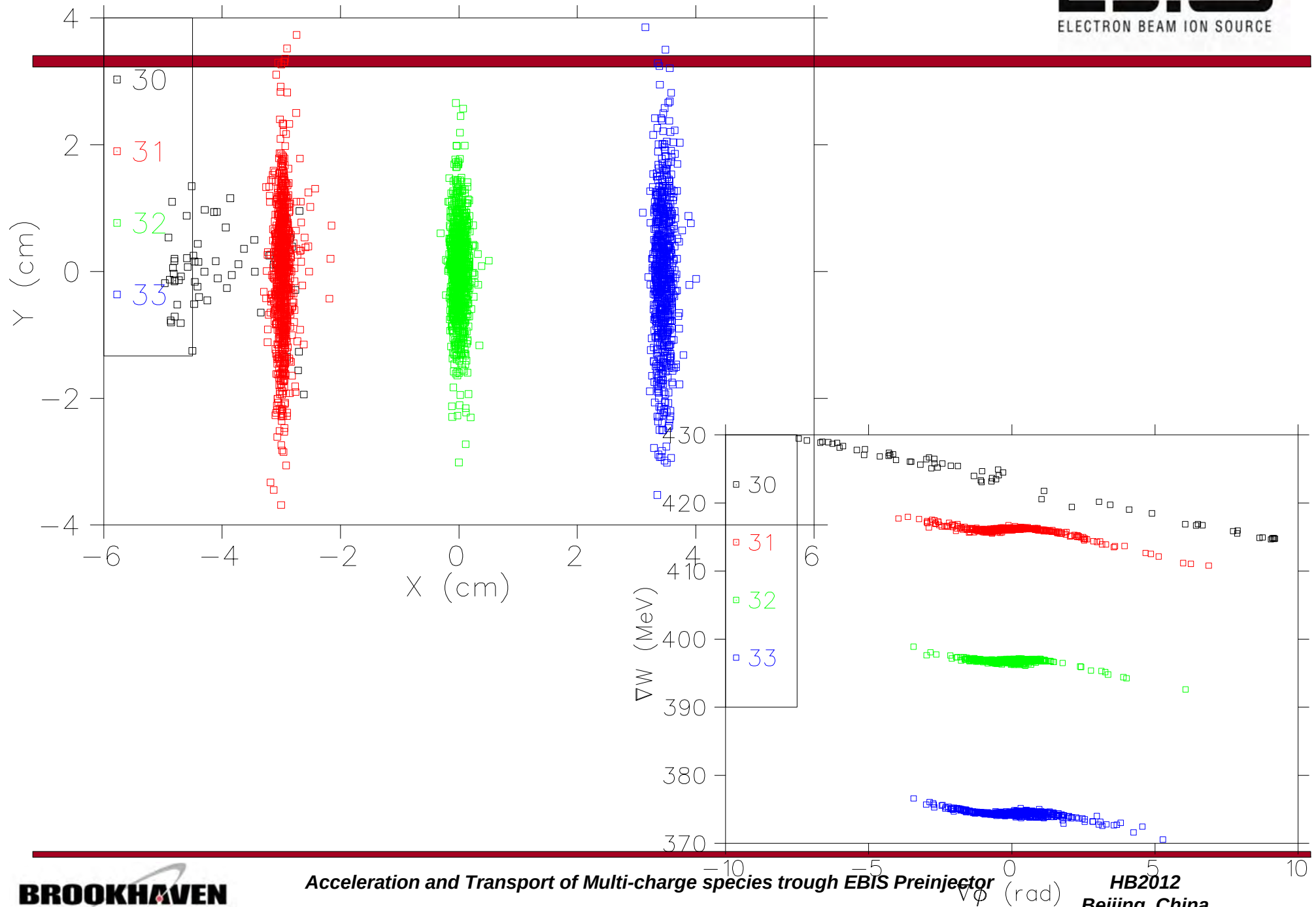


He¹⁺ – first beam from EBIS transported to Booster

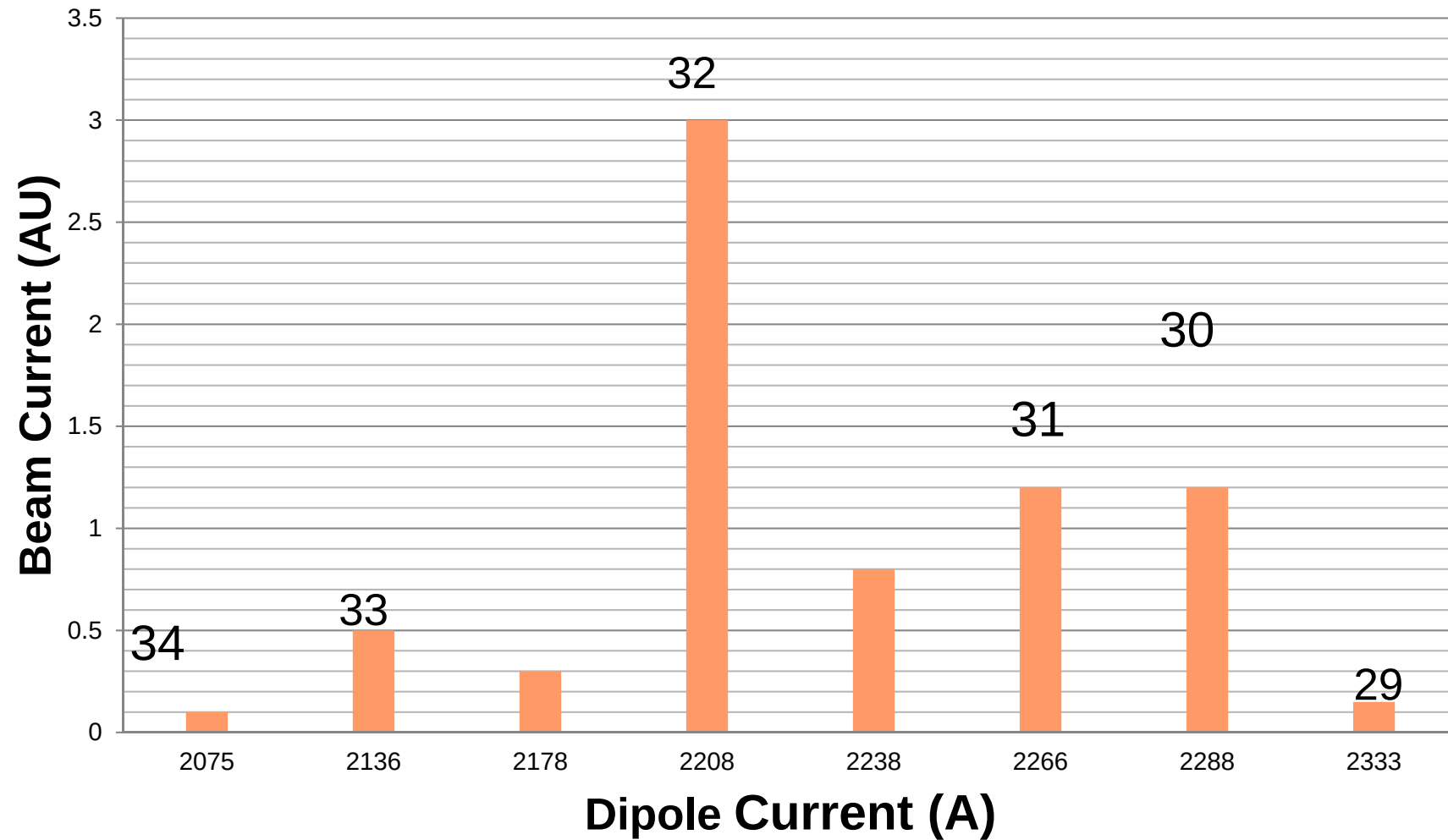


Green and pink traces are before and after the bends

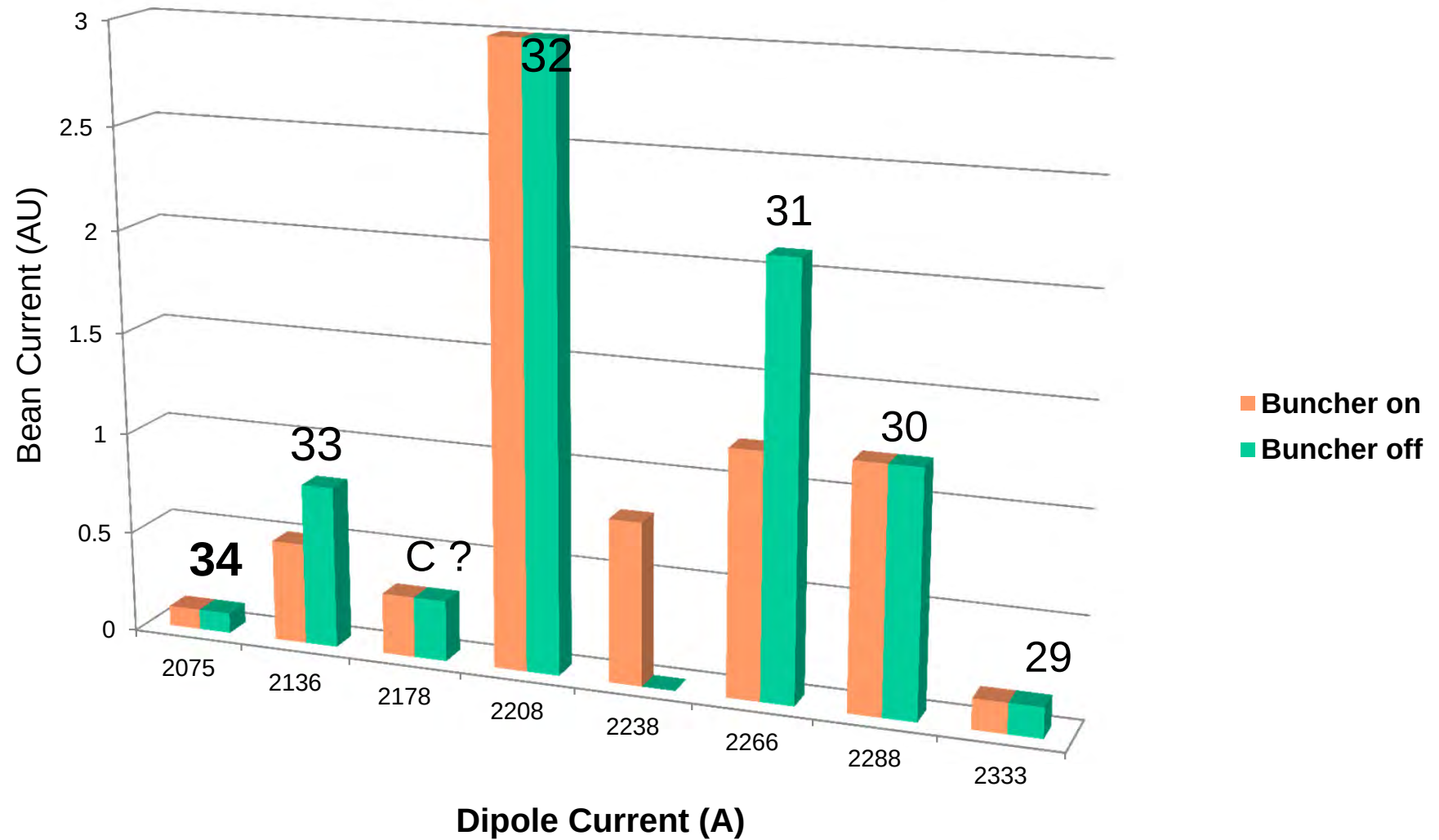
Middle of the Bend



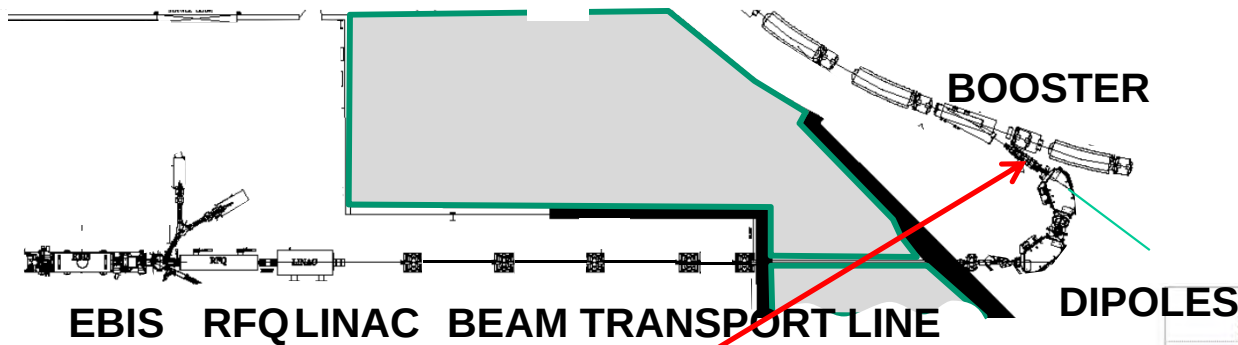
Gold at Middle of Bend



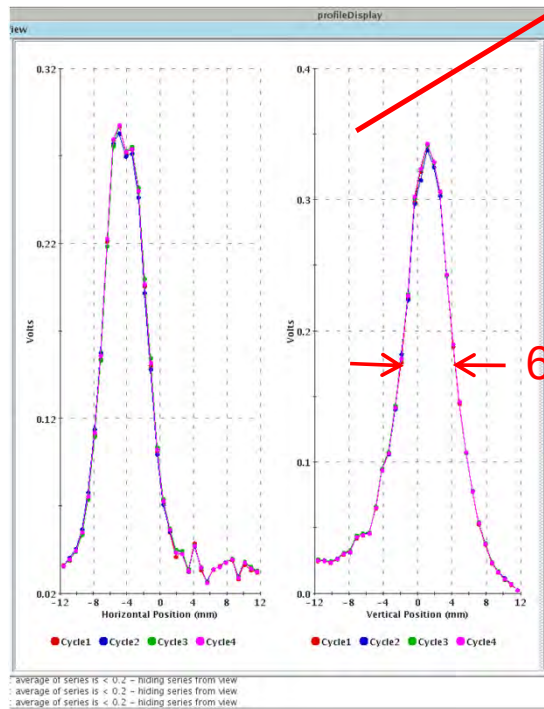
Gold at Middle of Bend: Buncher On/Off



EBIS Beam circulating in the Booster

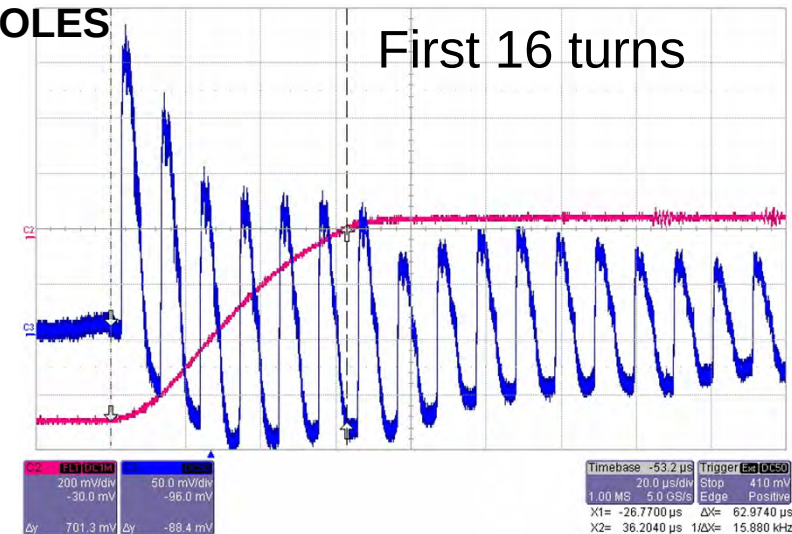


He⁺ beam from EBIS
circulating in the Booster



Profiles right
before the Booster
injection

6 mm



From the measure of the beam debunching time in Booster

$$\Delta p/p = 2.0 \times 10^{-4} \pm 1.0 \times 10^{-4}$$

(Requirement $\leq 5.0 \times 10^{-4}$)

EBIS beams transported to middle of bend (2 MeV/u)



He⁺¹	Conf. Time	2.5 ms
	RFQ Power	40 kW
	Linac Power	76 kW
	Pulse width	<20 μ s
	Intensity	7×10^{10} ions / pulse

Transmission (helium):

RFQ Input FC	1.4 mA (18.5 nC)	
RFQ Output FC	1.4 mA (15.0 nC)	T=100%, (T=81% charge)
XF14 (after linac)	1.1 mA	T=79%
XF 89 (before bend)	1.1 mA	T=100%
XF108 (after both bends)	1.1 mA	T=100%

Gold Transmission to booster injection

Charge	EBIS#	(Measu)	RFQ	Linac	Bend	B INPUT	
26	1.2	0.033622		0	0	0	0
27	7.5	0.044829		0	0	0	0
28	20	0.064041		0	0	0	0
29	45	0.092859	0.000464		0	0	0
30	80	0.108229	0.060933	0.044206	0.005698116		0
31	111	0.120077	0.107589	0.106748	0.105307397		0
32	124	0.128082	0.123727	0.123727	0.123599103	0.123599	
33	111	0.120077	0.096182	0.091499	0.085134486		0
34	80.5	0.112072	0.07139	0.062872		0	0
35	49	0.076049	0.028975	0.021598		0	0
36	25.5	0.048031	0.015754	0.008357		0	0
37	11.5	0.03202	0.005487		0	0	0
38	4.5	0.020013	0.003162		0	0	0
			1	0.513662	0.459008	0.123599	0.123599
	Sim (nC)		54.5	27.9945	25.01659	6.7362	6.7362
	Measu (nC)		54.5	29.6	16.2	7.4	5.8
	Ratio		1.000	1.05735	0.647	1.10	0.86

Electron current 7.5 A

Design

$0.81 \cdot .128 \cdot 54.5$

=5.7 nC

Achieved

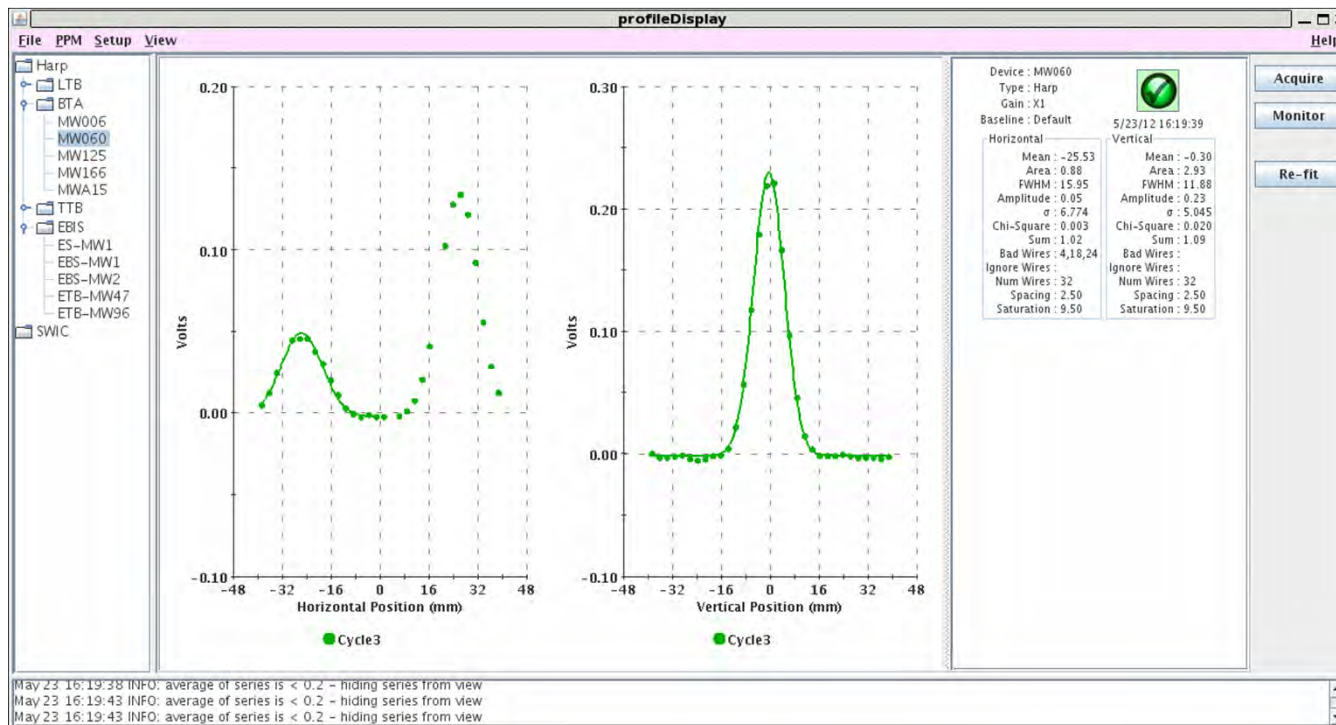
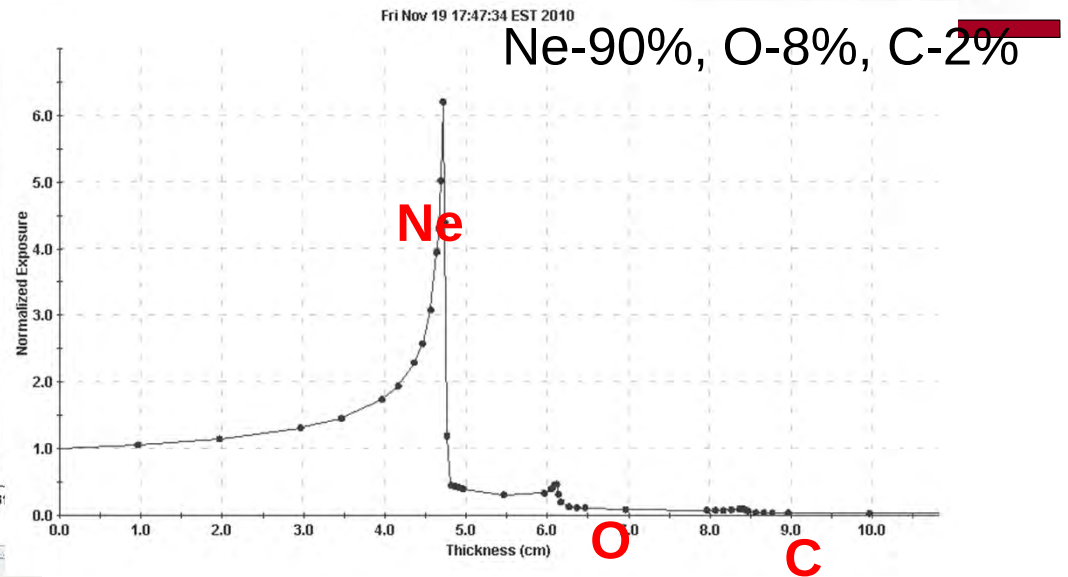
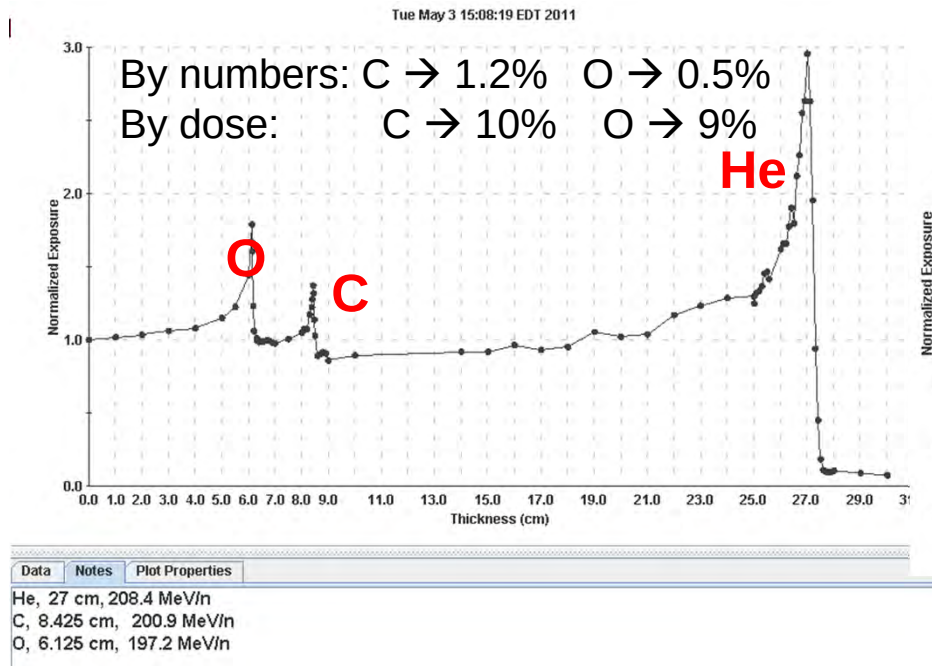
5.8 nC

Ratio

$5.8/5.7$

=1.02

Multiple ion Species seen after Booster



Beam profile in BTA

~ 60% Cu^{+11}

~ 40% Ar^{+7}

Tuning procedure with Multiple Charge Species (PLAN vs Real)



We have studied the tuning procedure for the linac. Summary as follows:

Set all the quad to calculated values and bunchers off

•LEBT

-With TOF maximize beam current for desired charge state (NOT practiced)

-Maximize beam current after RFQ to find the match into RFQ

•RFQ

-Since RFQ will be commissioned about 1.5 years earlier than linac, (NOT done)
we will set up analyzing magnet to set the correct amplitude (e. g Au^{+32})

•IH Linac

Maximizing the beam current for desired charge state using Booster
to set phase and amplitude of IH Linac

•MEBT

-Maximizing current after IH Linac, by tuning quads and buncher (using Booster)

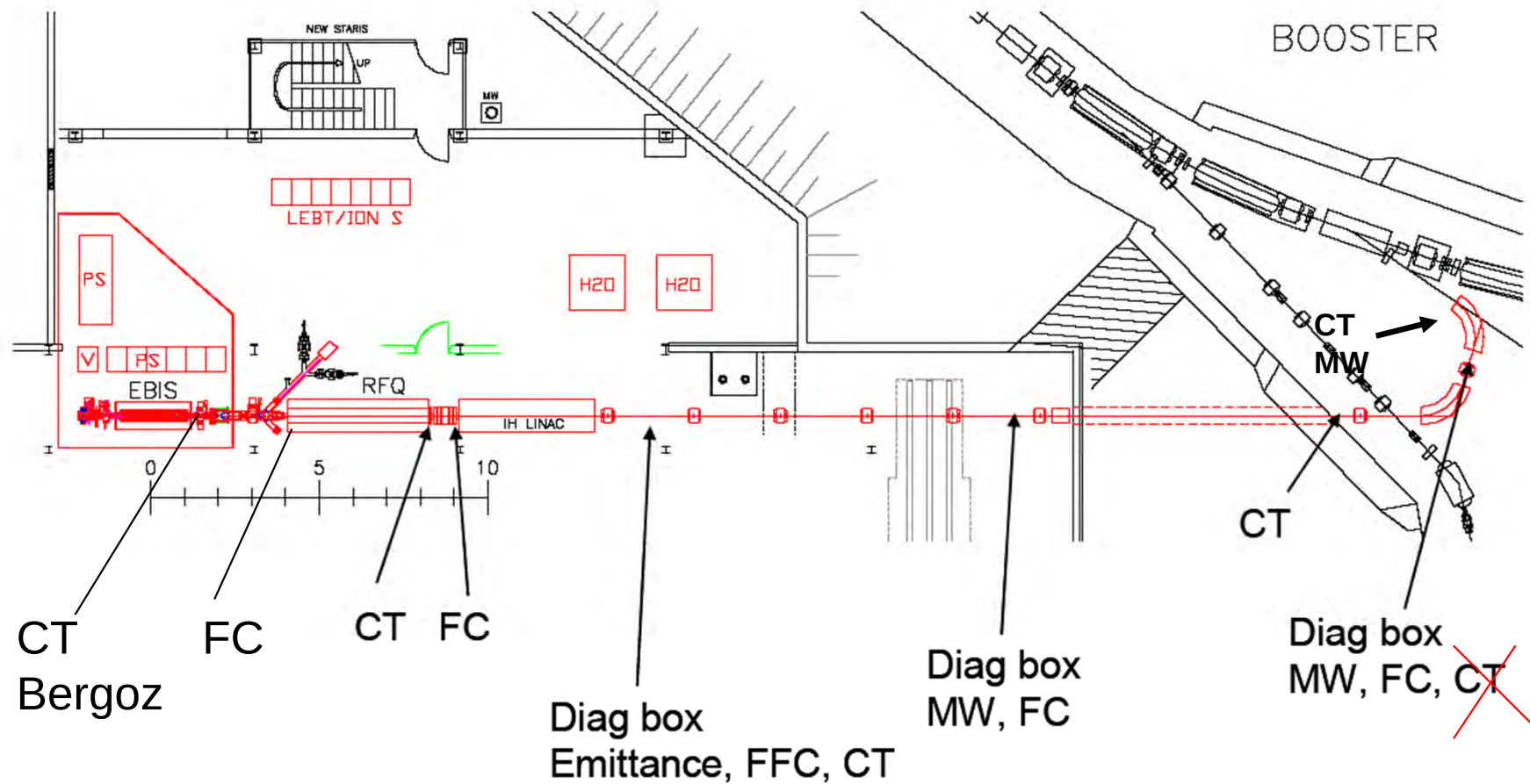
•HEBT

-Minimize the energy spread by tuning buncher amplitude and looking
beam size with profile monitor at high dispersion at the bend (using Booster)

-Verify bend magnet setting with Booster

Conclusions

- EBIS pre-injector provided Au and Cu beam for RHIC Run 12 and tens ions species to NSRL
- Acceleration and transport of multi ion species seen in the EBIS pre-injector and Booster
- EBIS pre-injector is very stable, reliable and reproducible



CT=current transformer; FC= Faraday cup; FFC=fast Faraday cup; MW=multiwire.

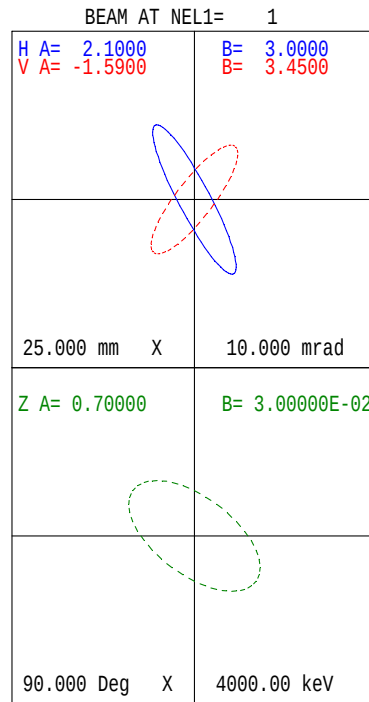
- **Exit of EBIS:** CT (Bergoz), depends on the voltage on nearby electrostatic grid lens
- **RFQ Input:** FC, diameter 13 mm, can collect +/- 30% $\Delta Q/Q$
- **RFQ Output:** CT, lot of noise due to RF of RFQ, and pulse quadrupole, NOT USED
- **MEBT Output:** FC, Not biased, electron suppression due to magnetic field of quad
- **Linac Output:** CT (XF14), sensitive to EBIS voltage configuration
 - **FFC:** due to multiple charge, not much used
- **Middle of HEBT:** FC47, diameter 30mm, not much used due to its size, beam diameter is much bigger
 - **MW47:** Used to determine emittance with multiple charge species

- Before the bend: CT, (XF89)
- Middle of the bend: FC96, 30 mm diameter, sensitive to ion species since biased is limited to few kV, read higher for lower q/m not used in transmission calculation
MW96: very useful, scanning magnetic field see all the ion species,
scrapers, Very important to stop other charge
- Input of booster: CT (XF108), very important to determine injection efficiencies
- Looking for diagnostics to distinguish charge states

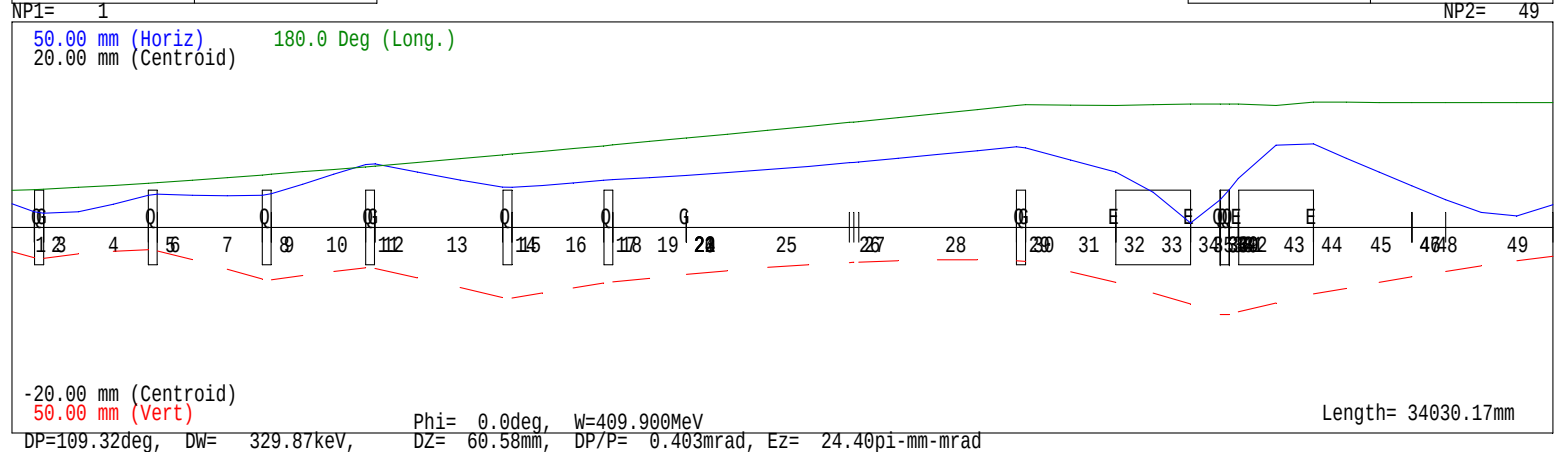
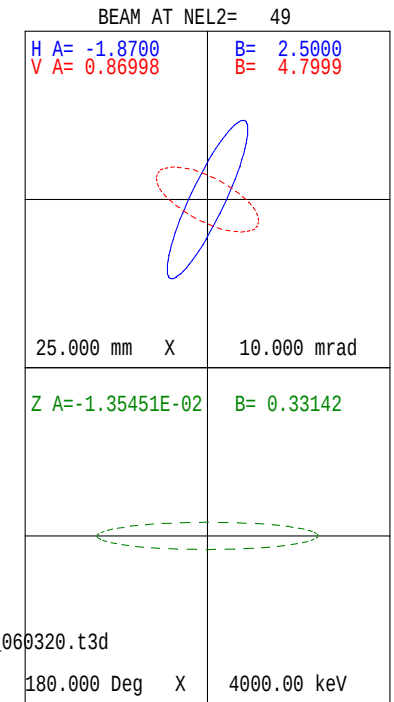
Optics of the Transport line to the Booster

Quad#	G T/m
1	3.8
2	2.9
3	2.3
4	2.3
5	1.6
6	0.3
7	1.5
8	2.2

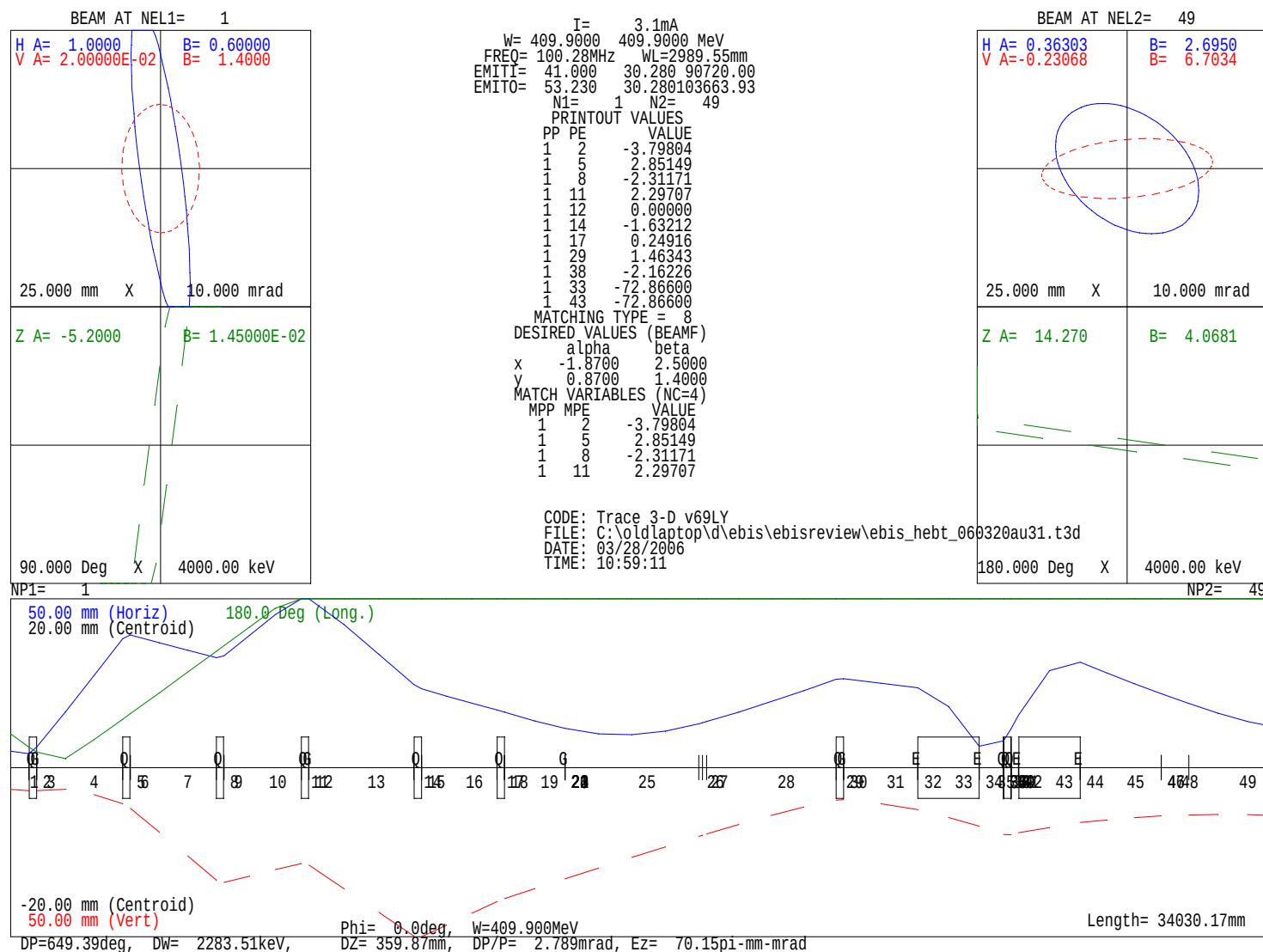
Buncher	V kV
1	0
2	33



I= 3.1mA
W= 409.9000 409.9000 MeV
FREQ= 100.28MHz WL=2989.55mm
EMIT= 11.000 10.280 35000.00
EMIT= 12.359 10.280 36056.97
N1= 1 N2= 49
PRINTOUT VALUES
PP PE VALUE
1 2 -3.79804
1 5 2.85149
1 8 -2.31171
1 11 2.29707
1 12 0.00000
1 14 -1.63212
1 17 0.24916
1 29 1.46343
1 38 -2.16226
1 33 -72.86600
1 43 -72.86600
MATCHING TYPE = 8
DESIRED VALUES (BEAMF)
alpha beta
x -1.8700 2.5000
y 0.8700 4.8000
MATCH VARIABLES (NC=4)
MPP MPE VALUE
1 2 -3.79804
1 5 2.85149
1 8 -2.31171
1 11 2.29707
CODE: Trace 3-D v69LY
FILE: C:\oldlaptop\d\ebis\ebisreview\ebis_hebt_060320.t3d
DATE: 03/28/2006
TIME: 10:00:04

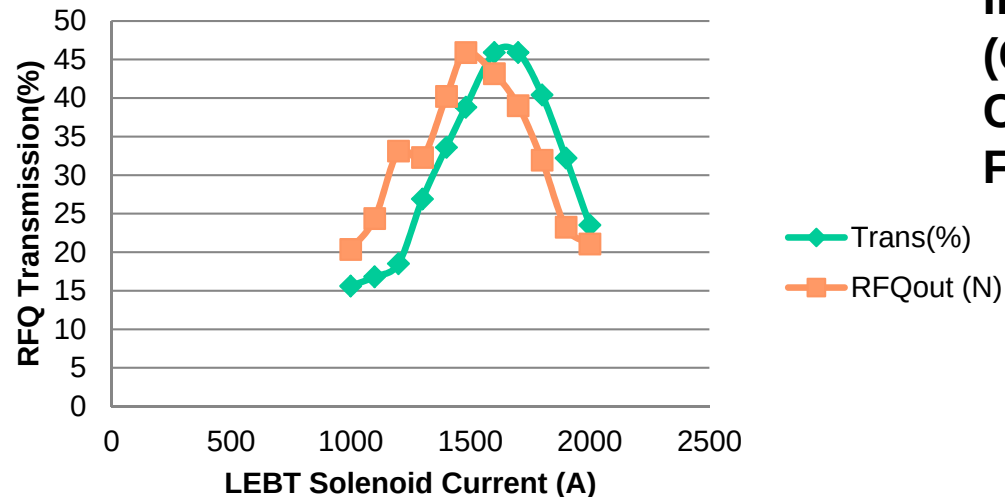


Beam Envelop for Au⁺³¹



Transmission of Au ions

RFQ Trans vs LEBT Solenoid



Simulation done using TRACK

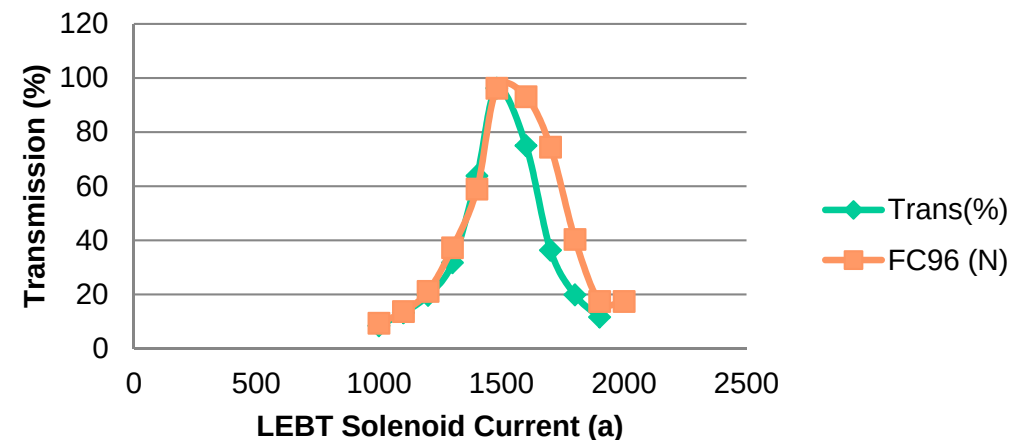
Input included Au^{32} and $\Delta p/p = \pm 10\%$

(C^2 , N^3 , O^3 , Ne^{3-4}) at EBIS voltage of 104 kV.

Current was measured at end of MEBT by Faraday cup

TRACK, PARMILA, and LORAS codes were used in simulations to transport Au^{+32} ions to the middle of the bend. Current was measured by Faraday cup (FC96) in the middle of the bend.

Au^{+32} Trans to Middle of the Bend vs LEBT Solenoid



Inflector Aperture

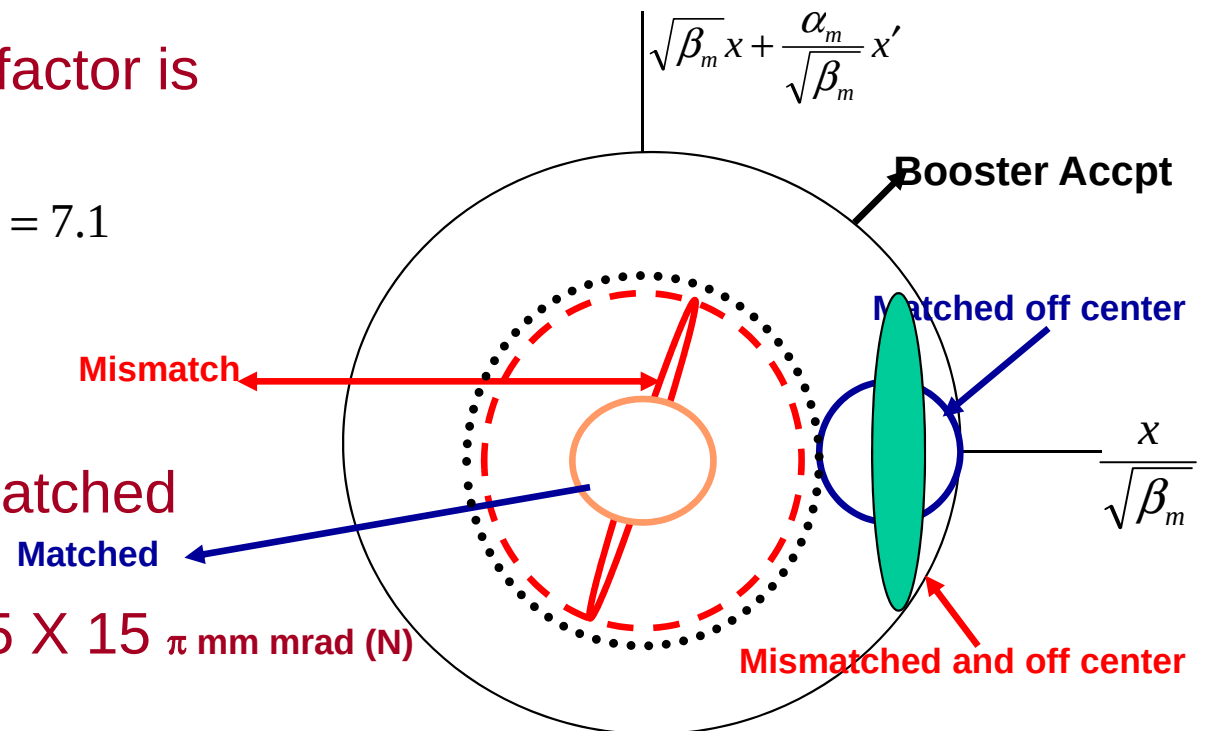
- Inflector gap is 17 mm and 2.2 meter long (**BNL #159, Gardner**)
=> Geometric acceptance 29π mm mrad, $\alpha_H = -1.87$, $\beta_H = 2.5$ m
Inflector acceptance 1.9π mm mr (N)
- Lattice function at exit of inflector are $\alpha_H = -1.72$, $\beta_H = 11$ m, $\eta_x = -2.4$ m

- Full beam size $2a = 2\sqrt{\epsilon_{90} \beta_H + (\eta_x \frac{\Delta p}{p})^2} = 22.1$ mm for matched injection

- Mismatch emittance dilution factor is

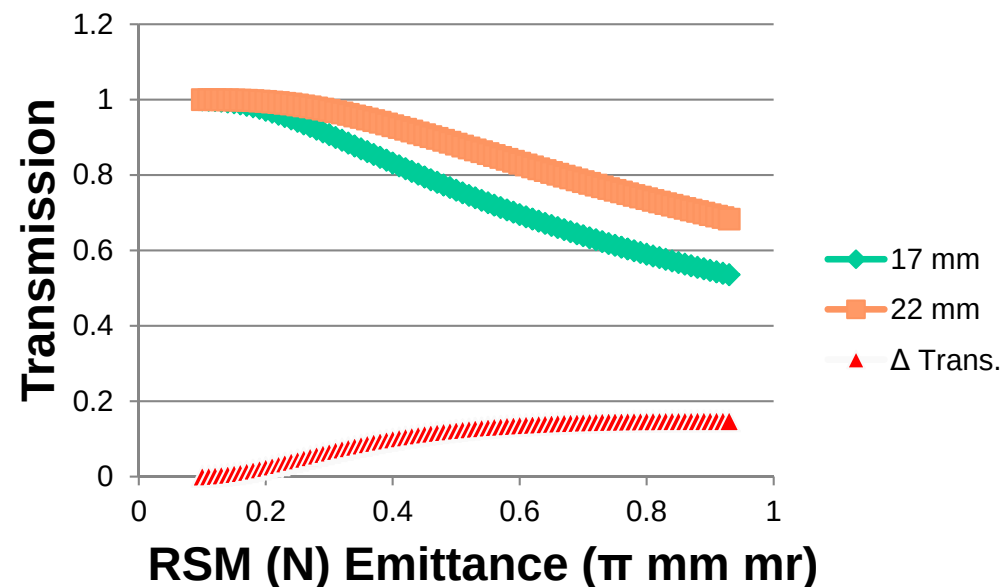
$$\frac{\epsilon_i}{\epsilon_m} = \frac{\sigma_i^2}{\sigma_m^2} = \frac{1}{2} (\beta_m \gamma_i + \beta_i \gamma_m - 2\alpha_i \alpha_m) = 7.1$$

- Linac emittance of 0.7(norm) will result in 5π mm mrad
R=15, 3.31 matched. 2.6 mismatched
- Incoherent tune shift 0.08
- Booster acceptance (VXH) $4.5 \times 15 \pi$ mm mrad (N)



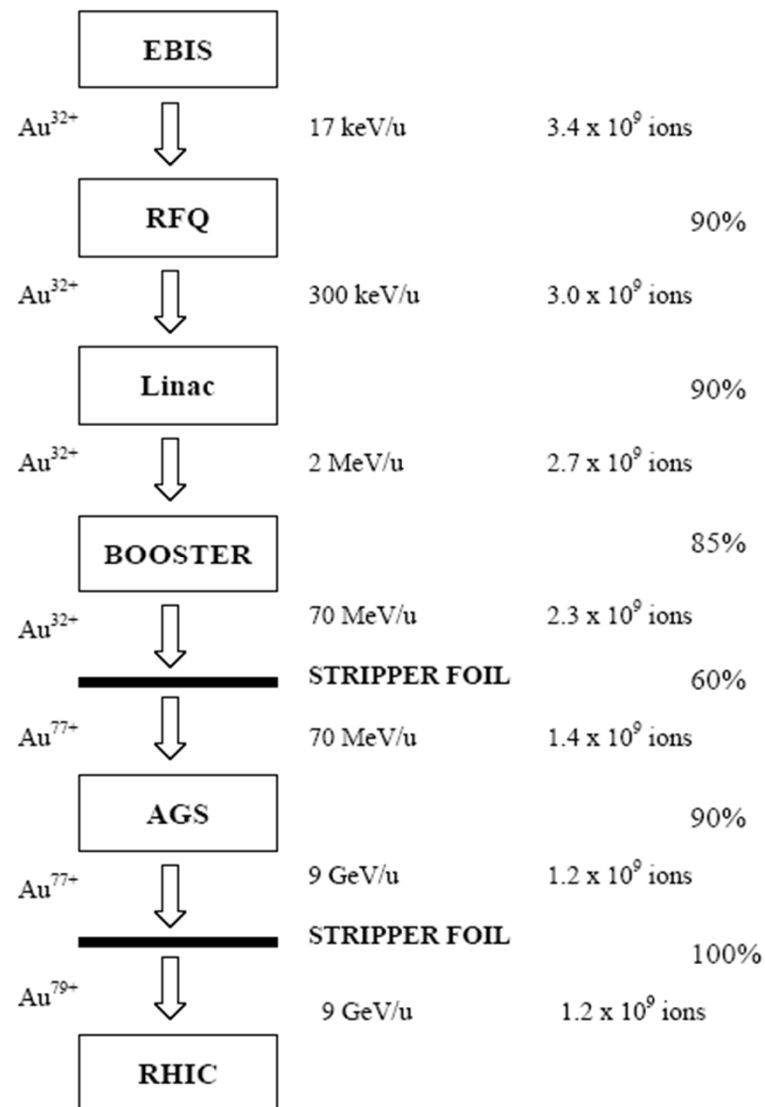
Emittance at Inflector

Last December inflector aperture was increase to 22 mm from 17 mm
The transmission was increase about 85% from 70% percent



=> Emittance (rms ,N) about 0.55 (π mm mr)

Ion/Pulse and Efficiencies for Au⁺³²



End-to-End Simulations (LEBT, Au⁺³²)

Pikin



2-D simulation with TRAK

This Plot Created On: March 24, 2006

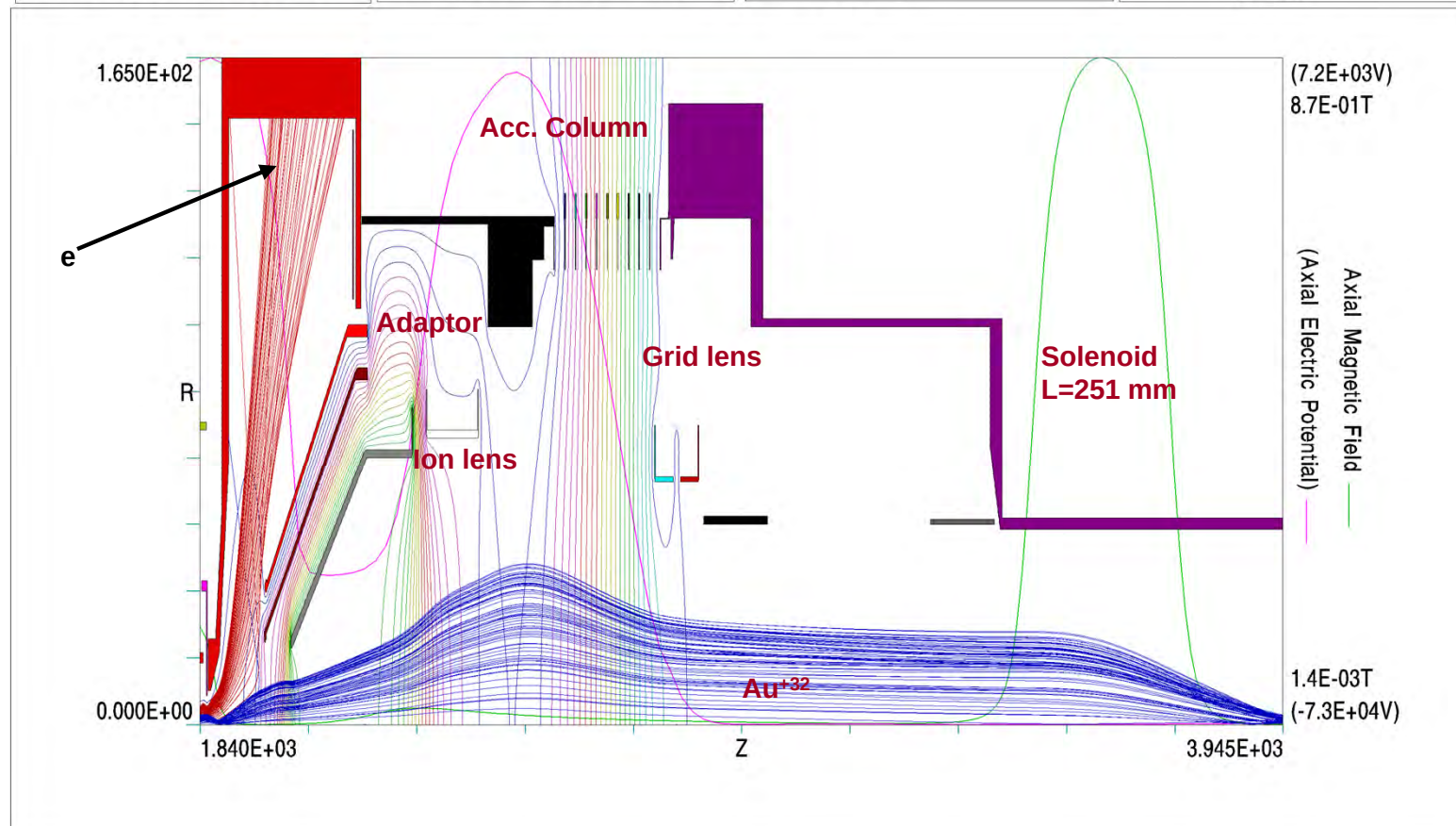
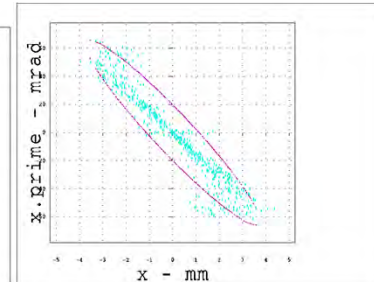
EOU File: FE55_0_67_500.EOU
BOU File: FBLEBT_6.BOU
TOU File: 55_0_67_1_78E5_2A.TOU

Optics parameters:

U_{ion lens} = -65 kV
U_{adaptor} = -10 kV
U_{grid} = +5.5 kV
IN_{magnet lens} = 1.78E+5 AmpxTurns

Beam parameters:

I_{el} = 10.0 A
I_{ion} = 8.0 mA
E_{RMS_norm_init} = 0.068 pi.mm.mrad
E_{RMS_norm_final} = 0.0803 pi.mm.mrad
Alfa_{final} = 1.01
Beta_{final} = 0.068
X_{final} = 2 mm
X'_{final} = 40 mrad



End-to-end Simulations

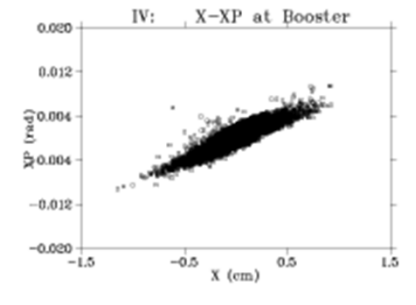
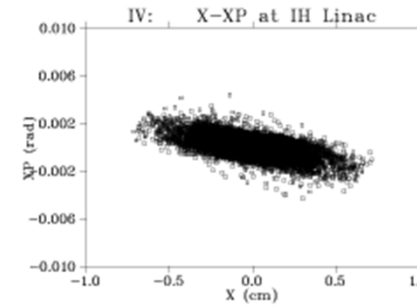
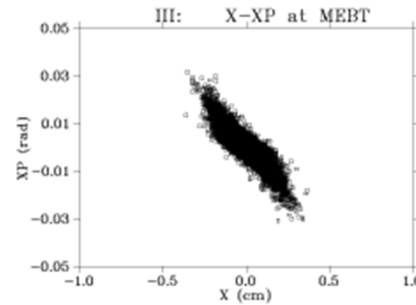
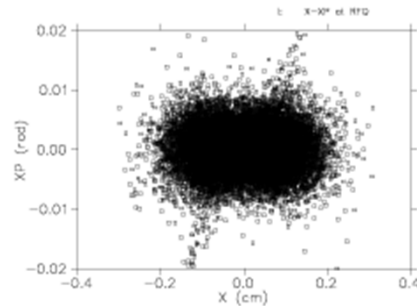
RFQ

MEBT

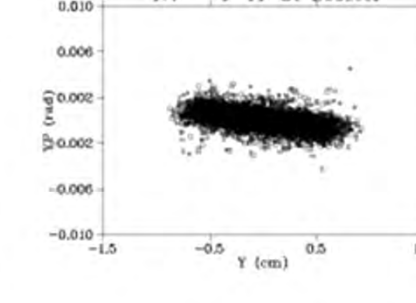
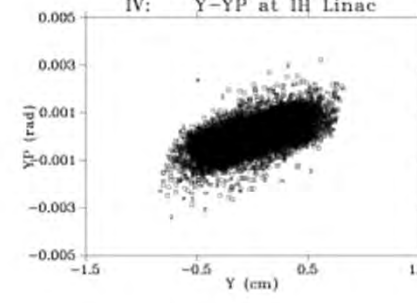
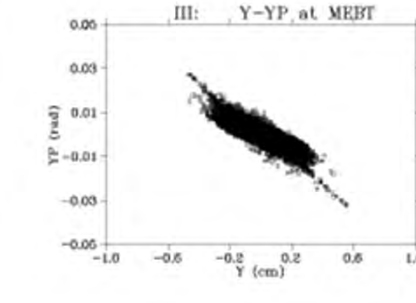
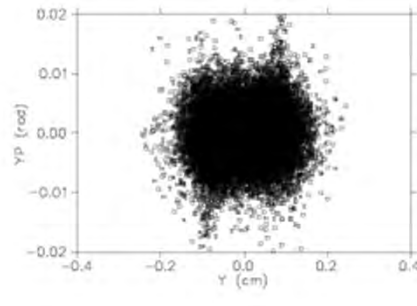
IH

HEBT

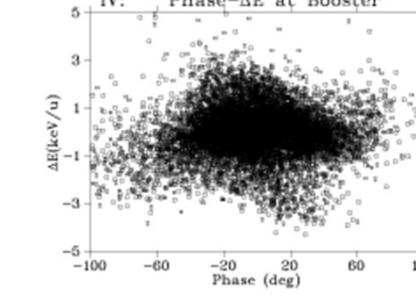
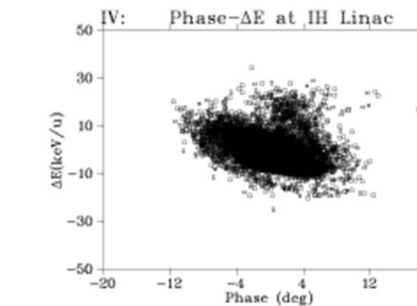
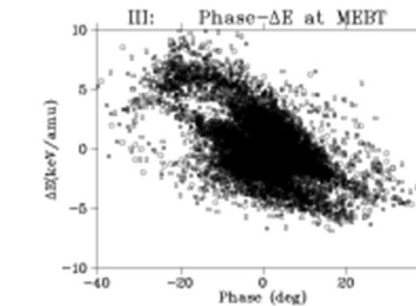
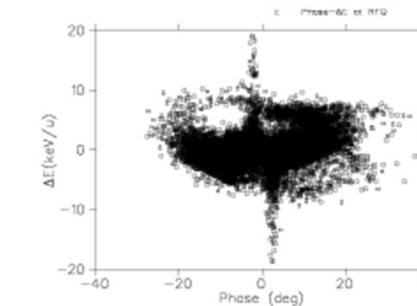
X-XP



Y-YP

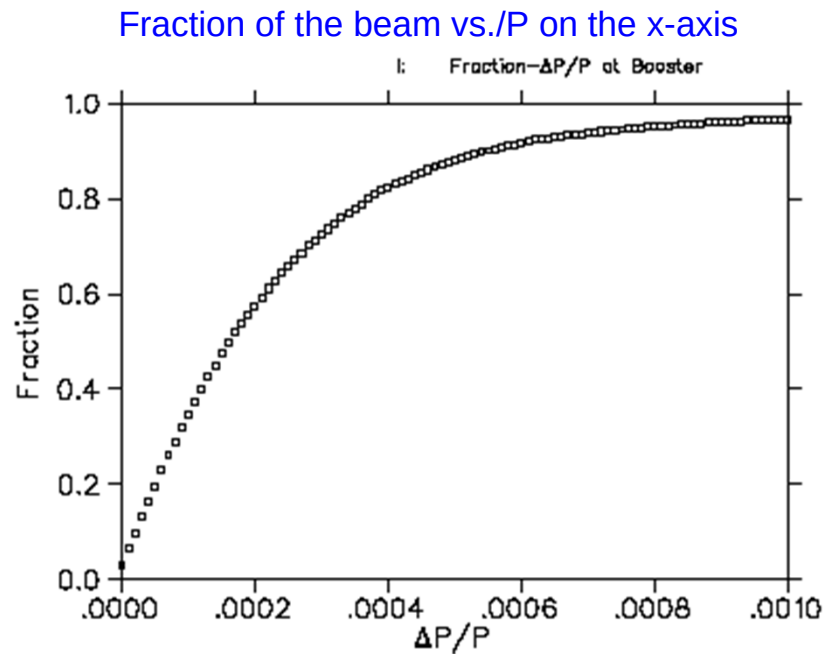


$\Delta W - \Delta\phi$



End-to-End Simulations

Transmission is defined with respect to the EBIS source



Location	ϵ (N, RMS)	
LEBT	$E_T (\pi \text{ mm mr})$	0.085
RFQ	$E_x (\pi \text{ mm mr})$	0.086
	$E_y (\pi \text{ mm mr})$	0.092
	$E_z (\pi \text{ ns-keV/u})$	0.0658
	Transmission	0.987
MEBT	$E_x (\pi \text{ mm mr})$	0.096
	$E_y (\pi \text{ mm mr})$	0.090
	$E_z (\pi \text{ ns-keV/u})$	1.425
	Transmission	0.982
IH-DTL	$E_x (\pi \text{ mm mr})$	0.097
	$E_y (\pi \text{ mm mr})$	0.096
	$E_z (\pi \text{ ns-keV/u})$	0.779
	Transmission	0.966
HEBT	$E_x (\pi \text{ mm mr})$	0.146
	$E_y (\pi \text{ mm mr})$	0.122
	$T.(\Delta p/P=0.1\%)$	0.960
	$T.(\Delta p/P=0.05\%)$	0.896

End-to-End Simulations with error tolerances



- Ion extractor $\pm 1\%$, Ion lens $\pm 0.5\%$, Adapter $\pm 1\%$, Gridded lens $\pm 0.2\%$, Platform $\pm 0.5\%$, Solenoid $\pm 0.2\%$
- Quad Alignment ± 0.1 mm (MEBT, IH-DTL, HEBT)
- Quad Strength $\pm 0.1\%$ (MEBT, IH-DTL, HEBT)
- Phase and Amplitude ± 0.5 deg, $\pm 0.5\%$ (RFQ, IH-DTL, Bunchers)
- Dipole Field Uniformity ± 0.1 % (x-y plane)
- Dipole Alignment ± 0.1 mm
- Harp 0.2 mm

End-to-End Simulations with Errors

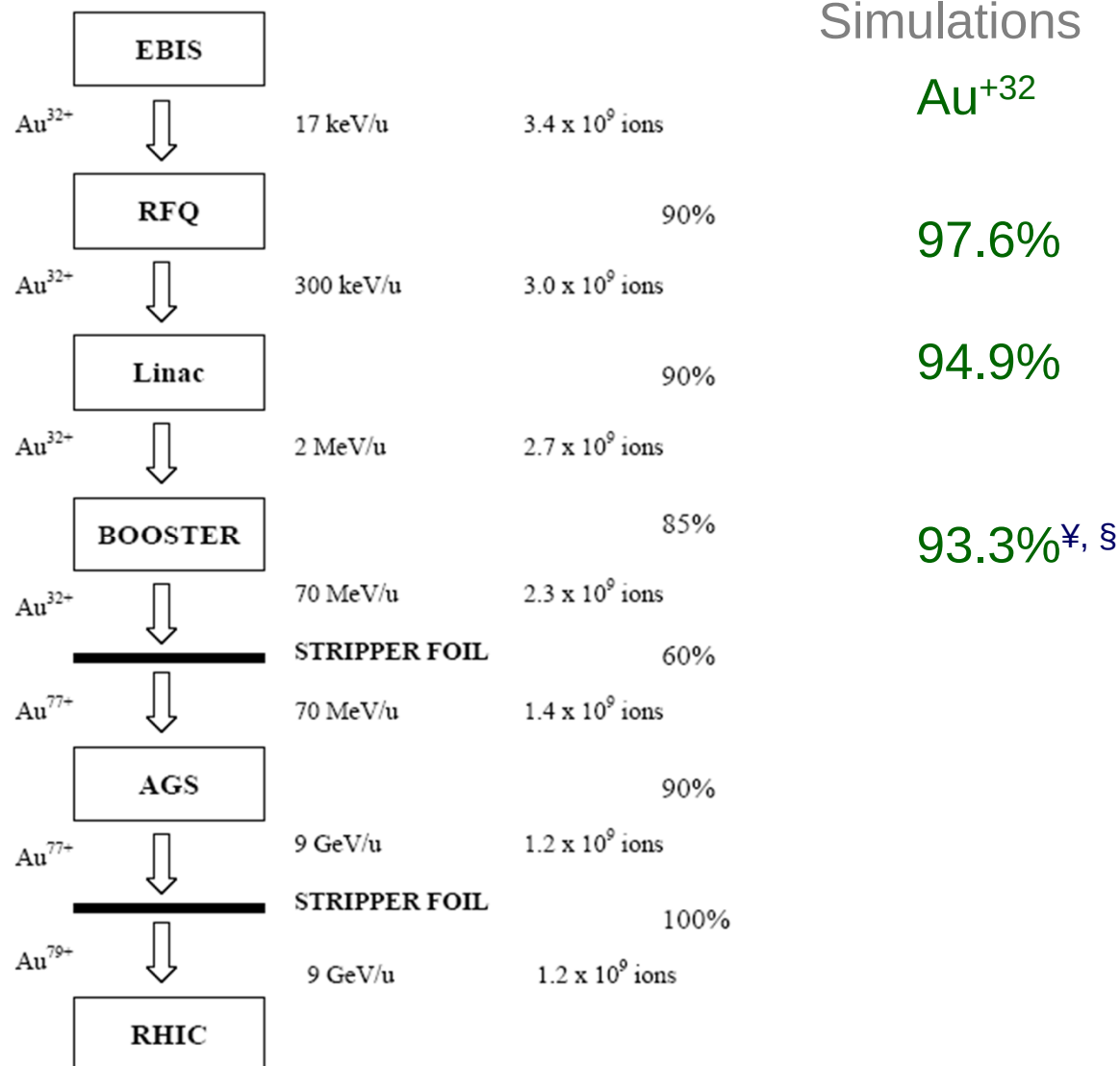
- 20 random seeds were used
- All the errors were uniformly distributed
- Simulation started at EBIS with 1000 micro-particles (TRAK 2D)
- At RFQ r-r' converted to x-x' and y-y' with >10000 micro-particles
- Alignment errors were corrected with automated steering scheme

End-to-End Simulations with Errors

- Transmission with respect to the EBIS Source
- Transmission for HEBT defines as particles with $\Delta p/p < \pm 0.05\%$

Location	Transmission (%)	ϵ (RMS, N) π mm mrad	ϵ (RMS, N) π mm mrad	ϵ (RMS) π ns-keV/u	MMF (x/y)
LEBT Average (STD)	100 (0.00)	0.0927 (0.001)	0.0897 (0.003)	0.1390 (0.002)	0.05/0.25 (0.04/0.02)
RFQ Average (STD)	97.6 (0.75)	0.1223 (0.020)	0.1160 (0.009)	0.8500 (0.010)	0.04/0.06 (0.03/0.03)
MEBT Average (STD)	96.7 (0.79)	0.1159 (0.004)	0.1102 (0.006)	0.8043 (0.079)	0.55/0.46 (0.12/0.12)
IH-DTL Average (STD)	92.6 (1.19)	0.1286 (0.008)	0.1380 (0.012)	1.4513 (0.328)	0.38/0.57 (0.09/0.12)
HEBT Average (STD)	86.4 (1.22)	0.1670 (0.008)	0.1480 (0.011)	0.8078 (0.054)	0.03/0.08 (0.04/0.18)

Ion/Pulse and Efficiencies for Au⁺³²



§ ΔP/P = 0.05%

¥ Longitudinal only

Performance Requirements of the EBIS

Species	He to U
Output (single charge state)	$\geq 1.1 \times 10^{11}$ charges
Intensity (examples)	3.4×10^9 Au ³²⁺ / pulse (1.7 mA) 5×10^9 Fe ²⁰⁺ / pulse (1.6 mA) 6.3×10^{10} He ¹⁺ / pulse (2.0 mA)
Q/m	≥ 0.16 , depending on ion species
Repetition rate	5 Hz
Pulse width	10 - 40 μ s
Switching time between species	1 second
Output emittance (Au ³²⁺)	$< 0.18 \pi$ mm mrad,norm,rms
Output energy	17 keV/amu
$\Delta P/P$	<0.05 %

Space Charge in LEBT

Current 10 mA

	Z	A	Q	Q/m	vext	EPS(un)	Perv(gen)	ET	SCT	Ratio	Debye Len
P	1	1	1	1.00	16.2	67	0.00296	0.0019	0.1971	104.5375	0.000436
He3	2	3	2	0.67	24.4	67	0.00197	0.0019	0.1314	69.69164	0.000534
D	1	2	1	0.50	32.5	67	0.00148	0.0019	0.0985	52.26873	0.000616
Si	14	28	12	0.43	37.9	67	0.00127	0.0019	0.0845	44.80177	0.000665
Au	79	197	32	0.16	100.0	67	0.00048	0.0019	0.0320	16.9807	0.001081

Measurement for 1.7 mA Au⁺²⁵ (n,rms) = 0.125 π mm mrad @ 20keV

Calculated(n,rms) = 0.125 π mm mrad

Envelope equation

$$R'' + k_0^2 R - \frac{(4\epsilon_{rms})^2}{R^3} - \frac{K}{R} = 0$$

Gen. Perv. (K) = $QI / (2 \pi \epsilon_0 m c^3 \beta^3 \gamma^3)$

Debye Length(λ_D) = $2 \epsilon^2_{rms} / K = (1/8)(R/\lambda_D)^2$

Energy = 17 keV/u
 $\beta = 0.006017$
 $\gamma = 1.000018$
R = 15 mm