EFFECTS OF POTENTIAL ENERGY SPREAD ON PARTICLE DYNAMICS IN MAGNETIC BENDING SYSTEMS *

R. Li , Jefferson Lab, Newport News, VA 23606, USA

Abstract

Understanding CSR effects for the generation and transport of high brightness electron beams is crucial for designs of modern FELs. Most studies of CSR effects focus on the impacts of the longitudinal CSR wakefield. In this study, we investigate the impact of the initial retarded potential energy of particles, due to bunch collective interaction, on the transverse dynamics of particles on a curved orbit. It is shown that as part of the remnants of the CSR cancellation effect when both the longitudinal and transverse CSR forces are taken into account, this initial potential energy at the entrance of a bending system acts as a pseudo kinetic energy, or pseudo energy in short, because its effect on particle optics through dispersion and momentum compaction is indistinguishable from effect of the usual kinetic energy offset from the design energy. Our estimation indicates that the resulting effect of pseudo energy spread can be measurable only when the peak current of the bunch is high enough such that the slice pseudo energy spread is appreciable compared to the slice kinetic energy spread. The implication of this study on simulations and experiments of CSR effects will be discussed.

INTRODUCTION

When a high brightness electron beam is transported through a curved orbit in a bending system, the particle dynamics is perturbed by the coherent synchrotron radiation (CSR) forces, or the collective Lorentz force as a result of the Lienard-Wiechert fields generated by particles in the bunch. The longitudinal CSR interaction takes place when the fields generated by source particles at bunch tail overtake the motion of the test particles [1] at bunch head and cause changes of kinetic energy for the head particles. For parameters currently used in most machine designs and operations, the approximation of the longitudinal CSR force by that calculated using 1D rigid-line bunch model [2] often gives good description of the observed CSR effects [3].

In addition to the longitudinal CSR force, the transverse CSR force [4] can directly perturb transverse particle dynamics. This force features energy independence and, due to divergent contribution from nearby-particle interaction, has strong nonlinear dependence on the transverse (and longitudinal) positions of particles inside the bunch. Meanwhile, the potential energy change, as a result of both longitudinal and radial CSR or Coulomb forces, can cause change of kinetic energy of the particles and impact transverse particle dynamics via dispersion. The joint effects of both the transverse CSR force and the kinetic energy change on bunch transverse dynamics have been analyzed earlier [5–7], and it is found that their harmful effects related to the potent feature of strong transverse dependence are cancelled. After the cancellation, the transverse dynamics of particles is perturbed by the remaining driving factors such as the effective longitudinal CSR force and the centrifugal force related to particles' initial potential energy.

In this study, we present the role of potential energy in the particle transverse dynamics after the cancellation effect is taken into account. We will show that the initial slice potential energy spread of a bunch, which we call pseudo slice energy spread, is indistinguishable from the usual slice kinetic energy spread in its perturbation to the transverse particle optics via both dispersion and momentum compaction. This effect is measurable only when the peak current of the bunch is high enough such that the pseudo slice energy spread is appreciable compared to the slice kinetic energy spread. The implication of this study on simulations and experiments of CSR effects will be discussed.

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ROLE OF POTENTIAL ENERGY IN BUNCH TRANSVERSE DYNAMICS

In this section the CSR cancellation effect is briefly reviewed. We show how a centrifugal force term, which is related to the initial potential energy of particles, emerges as one of the remnant of the cancellation. We also discuss the role of initial potential energy in transverse particle dynamics.

Consider an ultrarelativistic electron bunch moving on a circular orbit with design radius R and design energy $E_0 = \gamma_0 mc^2$. Let $x = r - R$ be the radial offset of particles from the design orbit. The single particle optics is determined by the configuration of the external magnetic fields, while for a bunch with high peak current, this design optics will be perturbed by the Lorentz force $\mathbf{F}^{\text{col}}$ resulting from the collective electromagnetic interaction amongst particles within the bunch. For transverse dynamics, such perturbation is expressed in terms of the first order equation

$$\frac{d^2x}{c^2dt^2} + \frac{x}{R^2} = \frac{\Delta E}{RE_0} + \frac{F_x^{\text{col}}}{E_0}, \tag{1}$$

with F_x^{col} being the radial component of the collective Lorentz force $\mathbf{F}^{\text{col}} = F_s^{\text{col}}\mathbf{e}_s + F_x^{\text{col}}\mathbf{e}_r$, and $\Delta E = E - E_0$ being the deviation of the kinetic energy from the design energy. The existence and effect of transverse CSR force F_x^{col} were first pointed out by Talman [4] when he analyzed F_x^{col} for space charge interaction of a bunch on a circular orbit using Lienard-Wiechert fields. His study shows that

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as a result of logarithmic divergence of nearby-particle interaction, this force is dominated by a term with the potent undesirable feature of strong nonlinear dependence over particles' transverse position within the bunch. Besides its role in the second driving term on the RHS of Eq. (1), the CSR force also causes change of particle's kinetic energy

$$\Delta E(t) = \Delta E\,(t=0) + \int_0^t \mathbf{F}^{\rm col}\cdot\mathbf{v}dt', \qquad (2)$$

which impacts the beam optics via dispersion as depicted by the first driving term in Eq. (1). As an example, the case of zero bunch charge corresponds to $F_x^{\rm col} = F_s^{\rm col} = 0$ and $\Delta E = E(0) - E_0$, when Eqs. (1) and (2) are reduced to equations for single particle dynamics. For the case of 1D CSR model as used in ELEGANT simulation, one has (1) $F_x^{\rm col} = 0$ and (2) $F_s^{\rm col}$ in Eq. (2) is obtained for a 1D bunch moving along the design orbit with its line-charge density distribution obtained by projecting the actual 3D bunch distribution (at the time of force calculation) onto the design orbit, with the assumption that this 1D projected distribution has been frozen as it is for all retarded times. In general, however, the kinetic energy can be changed by the usual longitudinal CSR force acting on the bunch as well as by the potential energy change caused by various ways of particle-bunch interaction. Examples of such particle-bunch interaction include (1) betatron motion in the potential well set by the radial CSR force for a coasting beam [8], (2) noninertial space charge force related to the radiative part of the longitudinal Lienard-Wiechert electrical field experienced by off-axis particles interacting with a line bunch on a circular orbit [9], (3) longitudinal space-charge interaction for a converging bunch on a straight section right before the bunch entering into the last dipole of a bunch compression chicane [10]. In all these examples, the collective-interaction-induced potential energy $e\Phi^{\rm col}$ shares the same feature of strong nonlinear dependence of particles' transverse position as that shown in the Talman's force.

The relation between the two driving terms in Eq. (1) becomes clear when both $F_x^{\rm col}$ and ΔE for the test particle are written in terms of retarded potentials [5]. The Taman's force is written as

$$F_x^{\rm col} = F_x^{\rm eff} + F^{\rm CSCF}, \qquad (3)$$

with the centrifugal space charge force $F^{\rm CSCF}$ term and the effective tranverse force $F_x^{\rm eff}$ term defined respectively as

$$F^{\rm CSCF} = \frac{e\beta_s A_s^{\rm col}}{r}, \qquad (4)$$

$$F_x^{\rm eff} = -e\left(\frac{\partial\Phi^{\rm col}}{\partial x} - \beta\cdot\frac{\partial\mathbf{A}^{\rm col}}{\partial x}\right) - e\frac{dA_x^{\rm col}}{cdt}. \qquad (5)$$

Meanwhile the change of kinetic energy for the test particle is

$$\Delta E(t) = \Delta E(0) - e[\Phi^{\rm col}(t) - \Phi^{\rm col}(0)] + \int_0^t F_v^{\rm eff}(t')\,cdt', \qquad (6)$$

with the longitudinal effective force term $F_v^{\rm eff}$ defined as

$$F_v^{\rm eff} = e\left(\frac{\partial\Phi^{\rm col}}{c\partial t} - \beta\cdot\frac{\partial\mathbf{A}^{\rm col}}{c\partial t}\right). \qquad (7)$$

For a bunch with phase space density distribution $f(\mathbf{r},\mathbf{v},t)$, the retarded potentials $(\Phi^{\rm col},\mathbf{A}^{\rm col})$ in the above expression are given by

$$\begin{aligned}\Phi^{\rm col}(\mathbf{r},t) &= e\int\frac{f(\mathbf{r}',\mathbf{v}',t')}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}'d\mathbf{v}',\\ \mathbf{A}^{\rm col}(\mathbf{r},t) &= e\int\frac{\mathbf{v}'f(\mathbf{r}',\mathbf{v}',t')}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}'d\mathbf{v}'\end{aligned} \qquad (8)$$

for $t' = t - |\mathbf{r}-\mathbf{r}'|/c$, and with N_e electrons in the bunch,

$$\int f(\mathbf{r},\mathbf{v},t)d\mathbf{r}d\mathbf{v} = N_e. \qquad (9)$$

The advantage of Eqs. (3)-(7) is that the potent term in $F_x^{\rm col}$ of Eqs. (1) and (3), contributed from the divergent local interactions, is now cleanly represented by the centrifugal space charge force $F^{\rm CSCF}$ in Eq. (4), leaving the remaining effective radial force $F_x^{\rm eff}$ free from the energy-independent local divergence of the order of $F^{\rm CSCF}$. Similarly, all the local-interaction contributions to $\Delta E(t)$ are summarized by $-e[\Phi^{\rm col}(t) - \Phi^{\rm col}(0)]$ in Eq. (6), leaving the remaining contributions from the effective longitudinal force $F_v^{\rm eff}$ also free from the energy-independent local divergence. Substituting Eqs. (3) and (6) into Eq. (1), one obtains the transverse dynamical equation

$$\frac{d^2x}{c^2dt^2} + \frac{x}{R^2} = \frac{\delta_{\mathcal{E}}(0)}{R} + \hat{G}^{\rm cor} \qquad (10)$$

with $\delta_{\mathcal{E}}(0)$ the relative *total* energy deviation from design energy

$$\delta_{\mathcal{E}}(0) = \frac{\mathcal{E}(0) - E_0}{E_0}, \quad \text{for} \quad \mathcal{E}(0) = E(0) + E_\phi(0) \qquad (11)$$

with

$$E_\phi(0) = e\Phi^{\rm col}(0), \qquad (12)$$

and $\hat{G}^{\rm cor}$ contains all the terms related to the interaction Lagrangian [7] that has negligible energy-independent local-interaction contributions

$$\hat{G}^{\rm cor} = \frac{1}{E_0}\left(\frac{1}{R}\int_0^t F_v^{\rm eff}(t')dt' + F_x^{\rm eff}\right) + G_{\rm res} \qquad (13)$$

with the residual of cancellation

$$\begin{aligned}G_{\rm res} &= -e\frac{\Phi^{\rm col}(t) - \beta_s A_s^{\rm col}(t)}{RE_0}\\ &\simeq -\frac{e^2}{R}\int\frac{\gamma_0^{-2} + \theta^2/2}{|\mathbf{r}-\mathbf{r}'|}f(\mathbf{r}',\mathbf{v}',t')d\mathbf{r}'d\mathbf{v}',\end{aligned} \qquad (14)$$

for $\theta = [s(t) - s'(t')]/R$ being the angular distance between the test particle at the observation time and the source particle at retarded time. As we can see, the terms with

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potent local-divergence contributions in both $F_r^{\rm col}/E_0$ and in $\Delta E/(RE_0)$ of Eq. (1) are collected in $G_{\rm res}$ in Eq. (14). Moreover, since $\theta \rightarrow 0$ when $\mathbf{r}' \rightarrow \mathbf{r}$, the local divergence of the two terms cancels, leaving the residual with no energy-independent contributions from local interaction for any bunch distribution, as indicated by Eq. (14) [7, 11, 12].

Unlike the usual longitudinal CSR force, which is the result of EM field generated by tail of the bunch overtaking the head of the bunch [1], analyses of the radial CSR force based on Lienard-Wiechert fields show [11, 13] that $F_x^{\rm col}$ is dominated by contributions from head-tail interaction that has a sudden turn-on behavior as the bunch moves from straight section onto a circular orbit. It can be shown the dominant head-tail contributions in $F_x^{\rm col}$ are all included in $F^{\rm CSCF}$ of Eq. (4), and it turns on suddenly when the bunch moves from straight section $r \rightarrow \infty$ to $r = R$. Likewise the head-tail contributions are equally important for $-e\Phi^{\rm col}(t)$ term in ΔE of Eq. (6). Since the perturbation of ΔE on optics in Eq. (1) is via the driving term $\Delta E/(E_0 R)$ in Eq. (1), its effect on the transverse dynamics also has a sudden turn-on behavior as the bunch enters from straight section ($R \rightarrow \infty$) to a circular arc (R finite). Consequently the two terms involved in the cancellation in Eq. (14) share common behavior and the residual of the cancellation is orders of magnitude smaller than each of the two terms [12].

As the two potent terms $-e\Phi^{\rm col}(t)/(RE_0)$ and $e\beta_s A_s^{\rm col}(t)/(RE_0)$—contained in the first and second driving terms of Eq. (1) respectively—are cancelled, a centrifugal force term $e\Phi^{\rm col}(0)/RE_0$ emerges as the remnant of the cancellation. The effect of this term on particle dynamics is the focus of this paper. One important observation [11, 13] is that similar to features of the centrifugal space charge force, $e\Phi^{\rm col}(0)$ in Eq. (11) also contains contribution from the head-tail particle interaction with local divergence, and its effect on the transverse dynamics has the sudden turn-on behavior which is not cancelled away. On the other hand, it was noted [6, 12] that since $\delta_H(0)$ in Eq. (10) acts as the initial relative energy offset, so unlike the $e\Phi^{\rm col}(t)$ term, here $e\Phi^{\rm col}(0)$ in Eq. (11) does not cause emittance growth when the bunch is transported through an achromatic bending system such as a magnetic bunch compression chicane.

Another residual term left from the cancellation is $\hat{G}^{\rm cor}$ in Eq. (10). It is dominated by effects from the longitudinal tail-head CSR interaction, for which 1D CSR model is usually a good approximation. Exceptional situations occur when the Derbenev's criteria [5] for the validity of 1D CSR model is violated. Such situations include CSR interaction during roll-over compression [14] and cases of CSR-induced microbunching instability when either the modulation wavelength is very short or the bunch transverse size is very large. The correct evaluation of retarded potentials/fields in such situations requires extra care in identifying the source particles by finding the intersection of the past light cone of test particles with the 2D/3D bunch distribution in the history of bunch motion [14].

Let $(\Delta x_c, \Delta x'_c, \Delta z_c, \Delta\delta_c)$ be the phase space perturbation generated by $\hat{G}^{\rm cor}$ in Eq. (10). Then after combining Eq. (10) (for $s \simeq ct$) with $dz/ds = -x/R(s)$, one finds that the initial kinetic energy offset and potential energy of a particle at the entrance of a bending system always work together, as the joint entity $\delta_{\mathcal{E}0}$, in causing transverse and longitudinal chromatic effects for single particle optics, namely,

$$\begin{pmatrix} x \\ x' \\ z \\ \delta_H \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & 0 & R_{16} \\ R_{21} & R_{22} & 0 & R_{26} \\ R_{51} & R_{52} & 1 & R_{56} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \\ z_0 \\ \delta_{H0} \end{pmatrix} + \begin{pmatrix} \Delta x_c \\ \Delta x'_c \\ \Delta z_c \\ \Delta\delta_c \end{pmatrix}, \tag{15}$$

for

$$\delta_{\mathcal{E}0} = \delta_{k0} + \delta_{\phi 0}, \tag{16}$$

with $\delta_{k0} = (E(0) - E_0)/E_0$ the usual relative kinetic energy offset, and $\delta_{\phi 0} = E_{\phi 0}/E_0$. The significance of the impact of $\delta_{\phi 0}$ on beam dynamics depends on the comparison of the initial potential energy spread with the initial kinetic energy spread. The discussion of Lorentz gauge as the natural choice of gauge for exhibiting the CSR cancellation effect can be found in Ref. [7].

THE SLICE TOTAL ENERGY SPREAD FOR A GAUSSIAN BUNCH

As a simplest example, we consider the impact of E_ϕ on the slice spread of total energy for an electron bunch with an idealistic cylindrically symmetric 3D Gaussian density distribution

$$P(r,z) = \frac{1}{(2\pi)^{3/2}\sigma_r^2\sigma_z} \exp\left(-\frac{r^2}{2\sigma_r^2} - \frac{z^2}{2\sigma_z^2}\right). \tag{17}$$

The bunch arrives at the entrance of a bending system at $t = 0$ (we drop t dependence in the following discussion) after moving ultrarelativistically on a straight path with $v_s = \beta c$ and $v_x = v_y = 0$. Here we have $\sigma_x = \sigma_y = \sigma_r$ and $r^2 = x^2 + y^2$. The probability distribution of particles over the total energy offset $\Delta\mathcal{E} = \Delta E + E_\phi$, with $\Delta E, E_\phi$ and $\Delta\mathcal{E}$ being random variables, is

$$P_{\mathcal{E}}(\Delta\mathcal{E}) = \int P_E(\Delta\mathcal{E} - E_\phi) P_{E_\phi}(E_\phi) dE_\phi, \tag{18}$$

where we assume a Gaussian distribution for the kinetic energy

$$P_E(\Delta E) = \frac{1}{\sqrt{2\pi}\sigma_E} \exp\left(-\frac{(\Delta E)^2}{2\sigma_E^2}\right). \tag{19}$$

In this section we summarize how the distribution of total energy $P_{\mathcal{E}}(\Delta\mathcal{E})$, for the central slice ($z = 0$) of the bunch, deviates from the usual kinetic energy distribution $P_E(\Delta E)$ as a result of the potential energy distribution $P_{E_\phi}(E_\phi)$ of the particles. We note that $P_{\mathcal{E}}(\Delta\mathcal{E}) = P_E(E)$ when the potential energy vanishes, i.e., $P_{E_\phi}(E_\phi) = \delta(E_\phi)$.

The potential energy distribution $P_{E_\phi}(E_\phi)$ can be uniquely determined from the probability distribution of

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particles in the bunch $P(\mathbf{r},t)$ and the dependence of potential energy on the particle spatial distribution $E_\phi(\mathbf{r},t)$. For our example of cylindrical Gaussian distribution, with $(\tilde{r},\tilde{z}) = (r/\sigma_r, z/\sigma_z)$ and $w = \tilde{r}^2$, the probability for particles to lie between w and $w + dw$ for the central $z = 0$ slice is deduced from Eq. (17)

$$P_w(w) = e^{-w/2}/2, \qquad \text{with} \qquad \int_0^\infty P_w(w)dw = 1. \quad (20)$$

The potential energy of a test particle at $(\mathbf{x},t)$

$$E_\phi(\mathbf{x},t) = e\int \frac{\rho(x_r, t - |\mathbf{x}-\mathbf{x}_r|/c)}{|\mathbf{x}-\mathbf{x}_r|}d^3\mathbf{x}_r \quad (21)$$

can be obtained [15] by applying Lorentz transformation on the scalar potential from the bunch comoving frame to the lab frame. For our example, with $\alpha = (\sigma_r/\gamma\sigma_z)^2$, this yields

$$E_\phi(r,z) = E_{\phi 0} f(\tilde{r},\tilde{z}), \qquad \text{with} \qquad E_{\phi 0} = mc^2 I_p/I_A \quad (22)$$

for the peak current $I_p = N_e ec/(\sqrt{2\pi}\sigma_z)$ and Alfven current $I_A = e/(r_e c) = 17$ kA, and

$$f(\tilde{r},\tilde{z}) = \int_0^\infty \frac{d\tau}{(1+\tau)\sqrt{1+\alpha\tau}} \exp\left(-\frac{\tilde{r}^2}{2(1+\tau)} - \frac{\tilde{z}^2}{2(1+\alpha\tau)}\right). \quad (23)$$

The dependence of the form function $f(\tilde{x},\tilde{y},\tilde{z})$ is shown in Fig. 1.

For the central slice at $z = 0$, we have from Eq. (22)

$$E_\phi(r,0) = E_{\phi 0}\, U(w) \quad (24)$$

for the normalized potential energy

$$U(w) = \int_0^\infty \frac{d\tau}{(1+\tau)\sqrt{1+\alpha\tau}} \exp\left(-\frac{w}{2(1+\tau)}\right). \quad (25)$$

Combining $U(w)$ with $P_w(w)$ in Eq. (20), one gets the probability for the value of potential energy of a particle to reside between $\mathcal{E}_\phi$ and $\mathcal{E}_\phi + d\mathcal{E}_\phi$

$$P_{\mathcal{E}_\phi}(\mathcal{E}_\phi)\, d\mathcal{E}_\phi = P_U(U)dU = P_w(w)dw \quad (26)$$

or

$$P_U(w) \equiv P_{E\phi}(\mathcal{E}_\phi)\mathcal{E}_{\phi 0} = \frac{P_w(w)}{|dU(w)/dw|}. \quad (27)$$

The semi-analytical results of probability distribution P_U on U is then obtained from the parametric dependence of $(U(w), P_U(w))$ on w [15], as shown in Fig. 2, which automatically satisfies $\int P_U(U)dU = 1$. The probability of distribution for the total energy of particles in Eq. (18) then becomes

$$P_{\mathcal{E}}(\Delta\mathcal{E}) = \int P_E(\Delta\mathcal{E} - E_{\phi 0}U)P_U(U)dU. \quad (28)$$

For the example in Fig. 1, the average and rms of particle distribution over U are respectively $< U >= 15.0$ and $\sigma_U =$ 0.51. An estimation of the rms spread of the total energy resulted from Eq. (18) is given by

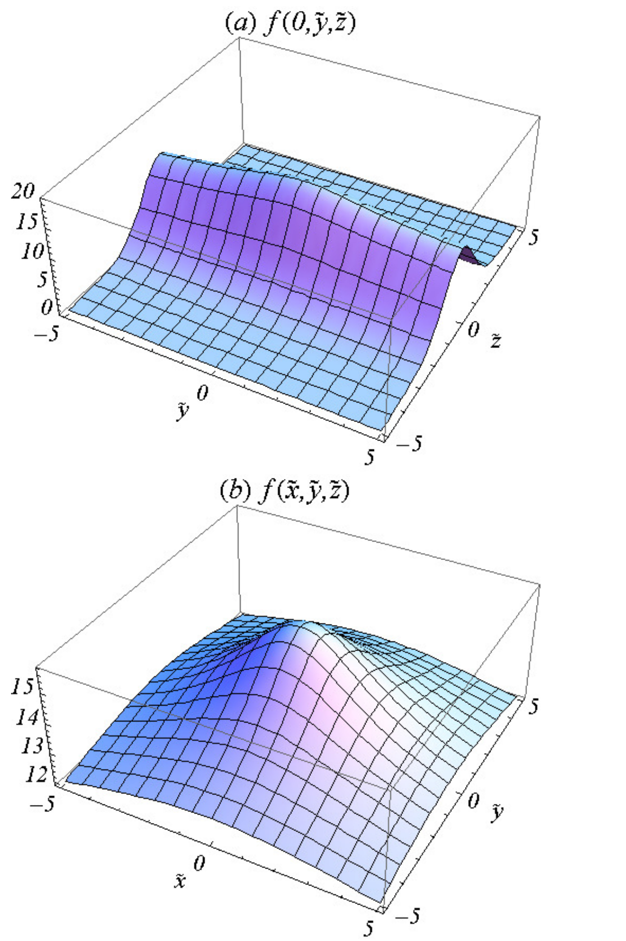

Figure 1: Behavior of $f(\tilde{x},\tilde{y},\tilde{z})$ in Eq. (23) for a cylindrical bunch over (a) the $\tilde{x} = 0$ plane and (b) the $\tilde{z} = 0$ plane for $\alpha = 5.7 \times 10^{-7}$.

$$\sigma_{\mathcal{E}} \simeq \sqrt{\sigma_E^2 + (E_{\phi 0}\sigma_U)^2}. \quad (29)$$

This relation shows that the rms of the total energy can be appreciably larger than that of the kinetic slice energy spread when

$$\xi \equiv \frac{E_{\phi 0}\sigma_U}{\sigma_E} \geq 1 \quad (30)$$

implying simutaneously both high peak current I_p and small kinetic energy spread σ_E.

To quantitatively compare the slice potential energy spread with the usual slice kinetic energy spread, and in particular, to compute the slice spread of the *total* energy, here we use the following parameters for the bunch [16, 17]

$$E_0 = 135\text{MeV},\ \sigma_z = 750\mu\text{m},\ \epsilon^n_{x,y} = 1\mu\text{m},\ \beta_{x,y} = 6\text{ m}, \quad (31)$$

which corresponds to $\alpha = 5.7 \times 10^{-7}$. We further choose $I_p = 120$ A and thus $E_{\phi 0} = 3.6$ keV. Substituting the semi-analytical results $P_U(U)$ as shown in Fig. 2 into Eq. (28), one gets the distribution of total energy spread $P_{\mathcal{E}}(\Delta\mathcal{E})$ for cases

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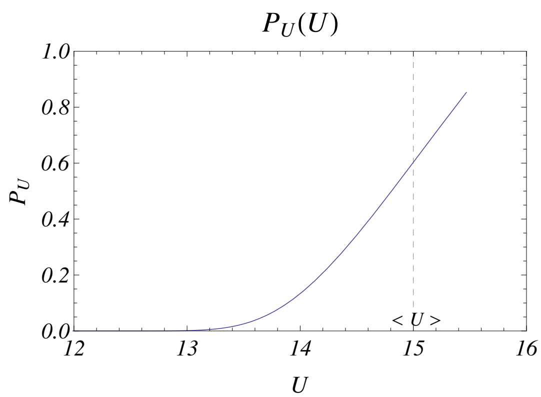

Figure 2: Probability distribution $P_U(U)$ for the central slice of a cylindrical Gaussian bunch with $\alpha = 5.7 \times 10^{-7}$.

when the slice kinetic energy spread takes (1) the typical value $\sigma_E = 3$ keV [18] when $\xi = 0.6$ and (2) $\sigma_E = 1$ keV when $\xi = 1.8$. The final results are shown in Fig. 3. It confirms our expectation that when $\xi \leq 1$ the potential energy has negligible effect on the slice total energy spread $\sigma_{\mathcal{E}}$, as displayed by Fig. 3a, yet when $\xi \geq 1$ the potential energy spread can cause appreciable widening of the total energy spread, as indicated by Fig. 3b in which the red curve for $P_{\mathcal{E}}(\Delta\mathcal{E})$ is much wider than the green curve for $P_E(E)$. Numerical evaluation using $P_{\mathcal{E}}(\Delta\mathcal{E})$ of Eq. (28) yields $\sigma_{\mathcal{E}} = 2.09$ keV for Fig. 3b, which agrees well with the estimation by Eq. (29) for $\sigma_E = 1$ keV. The semi-analytical results of $P_{\mathcal{E}}(\Delta\mathcal{E})$, shown by the red curves in Fig. 3, are also in good agreement with results from the Monte Carlo approach presented by the red dots in Fig. 3. Here for the Monte Carlo approach we populate $N = 10^4$ particles with random Gaussian distributions in both the 2D configuration space (in the $z = 0$ plane) and in the kinetic energy offset ΔE. One then evaluates the total energy $\Delta\mathcal{E}^i = \Delta E^i + E^i_\phi$ of the i-th ($i = 1, N$) particle by using E^i_ϕ in Eq. (22), and further obtain the histogram of particle distribution in $\Delta\mathcal{E}$.

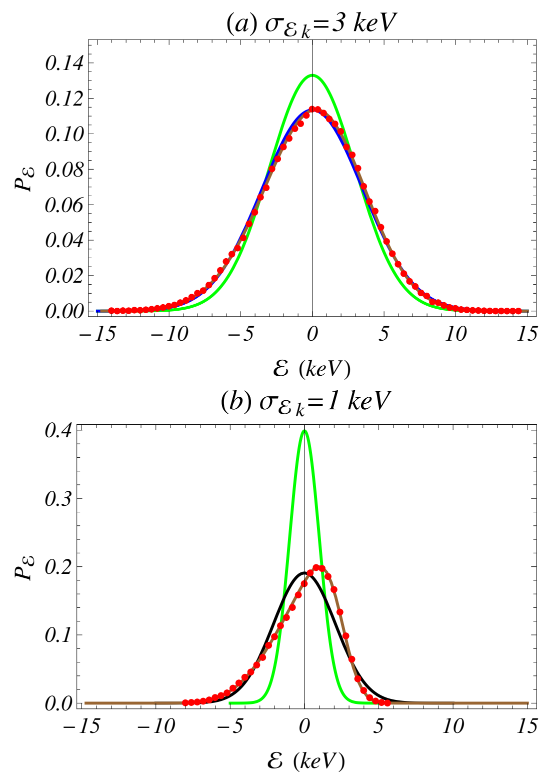

Figure 3: $P_{\mathcal{E}}(\Delta\mathcal{E})$ for $z = 0$ slice of a cylindrical Gaussian bunch with $E_{\phi 0} = 3.6$ keV, $\alpha = 5.7 \times 10^{-7}$ for (a) $\sigma_E = 3$keV and (b) $\sigma_E = 1$ keV. Green line: Gaussian distribution for kinetic energy $P_E(E)$; Brown line: semi-analytical results for distribution of total energy $P_{\mathcal{E}}(\Delta\mathcal{E})$; Solid black line: Gaussian distribution using the estimated rms in Eq. (29); Red dots: Monte Carlo results of $P_{\mathcal{E}}(\Delta\mathcal{E})$.

DISCUSSIONS

In this paper, after a brief review of the cancellation effect in the CSR-induced perturbation on bunch transverse dynamics for an ultrarelativistic electron bunch moving through a magnetic bending system, it is shown how, after cancellation, the effective CSR forces and the potential-energy related centrifugal force emerge as the net driving factors for particle transverse dynamics. The behavior of longitudinal effective CSR force for an energy-chirped Gaussian bunch has been studied earlier [14], and in this presentation we summarized that role and behavior of the initial potential energy term. The main conclusion is that the initial potential energy always works together with the initial kinetic energy in perturbing the particle transverse dynamics, and for the example of a Gaussian bunch, we find that the potential energy effect is important only when the peak current of the bunch is high and the slice kinetic energy spread is small, i. e., $\xi \geq 1$. Note that ξ in Eq. (30) should remain approximately a constant when the bunch gets compressed by a magnetic chicane with compression factor C, since both $E_{\phi 0}$ in Eq. (22) and the slice kinetic energy spread σ_E are increased by C. The potential energy spread for a general 3D bunch distribution requires careful numerical calculations.

For beam and machine parameters used in present designs, the 1D CSR model adopted by ELEGANT simulation often gives results in good agreement with measured CSR effects [3]. From the point of view of cancellation effect in CSR, the explanation of such success resides in the fact that in the 1D model both parts of the cancellation—the radial CSR force $F_r^{\rm col}$ and the potential energy change $-e[\Phi^{\rm col}(t) - \Phi^{\rm col}(0)]/R$—-are set to be zero, and only the impact resulted from the dominant longitudinal CSR force $F_v^{\rm eff}$ is being kept. The 2D/3D CSR effects are expected to show up in experiments for the unusual cases when the bunch distribution in x-z plane does not satisfy the Derbenev criteria $\sigma_x/(\sigma_z^2 R)^{1/3} \ll 1$, thence the behavior of $F_v^{\rm eff}$ will deviate [14] from that for a 1D rigid-line bunch, and when the bunch parameters are such that Eq. (30) is satisfied, thence the effect ot potential energy will appear as enlargement of the measured slice energy spread or lengthening of bunch

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length for a bunch at maximum compression as compared to results from the 1D CSR model.

Correct modeling of the cancellation effect poses significant challenge for 2D/3D CSR simulations, since it requires the two parts involved in the cancellation, $-e[\Phi^{\mathrm{col}}(t) - \Phi^{\mathrm{col}}(0)]/R$ and the radial CSR force F_r^{col}, be calculated with the same accuracy. Because $e[\Phi^{\mathrm{col}}(t) - \Phi^{\mathrm{col}}(0)]$ is an integrated effect of longitudinal and transverse CSR force on the circular orbit or Coulomb forces on a straight path, the cancellation effect can be taken care automatically only when both the CSR and space charge interaction are fully taken into account and the dynamics are advanced self-consistently. In addition, all the potential terms $\Phi^{\mathrm{col}}(t)$, $\Phi^{\mathrm{col}}(0)$ and $A_s^{\mathrm{col}}(t)$ have sensitive dependence on the 3D bunch density distribution, as shown in Fig. 1. Without adequate care, incomplete modeling could result in partial or none cancellation and cause artificial errors. More detailed discussions can be found in Ref. [15].

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