# Electromagnetic radiation in accelerator physics

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# Outline of the talk

- Introduction, order of magnitude estimates
- Time domain and spectral analysis of synchrotron radiation
- Undulator radiation
- Longitudinal and transverse formation lengths of radiation
- Fluctuations and correlations in radiation
- Radiative reaction force (aka CSR wake)
- Transition radiation

References in this tutorial are incomplete and may reflect the personal bias of the author.

# Maxwell's equations

Solving Maxwell's equations with (relativistic) equation of motion for charged particles covers most of applications in accelerator physics.

$$\nabla \cdot \mathbf{E} = 4\pi\rho \qquad \text{(I use CGS units, } \mathbf{E} \text{ and } \mathbf{B}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \text{have the same dimension)}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$$

This, however, is not an easy task. Computer codes are handy for solving many practical problems, but usually do not add much to understanding.

There are also quantum effects in radiation which require QED; they are beyond the scope of this lecture.

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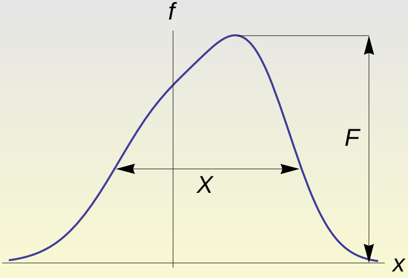
The art of OME is based on "fuzzy" logic. If we have two quantities, $a$ and $b$, then either $a \sim b$, or $a \gg b$, or $a \ll b$.




A function $f(x)$ is characterized by its height $F$ and the width $X$. We have

$$\frac{df}{dx} \sim \frac{F}{X}, \quad \int f(x)dx \sim FX.$$

Characteristic spectrum of $f$ contains harmonics with wavenumbers $k \sim X^{-1}$.

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In this tutorial I will mostly use OME instead of exact equations.

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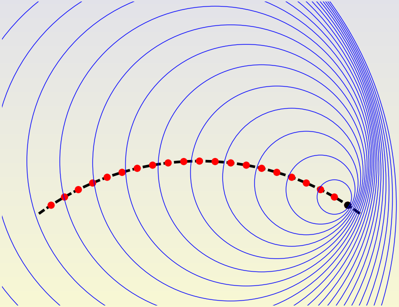
- acceleration, both along and perpendicular to $v$
- presence of material boundaries
- combination of the above

Examples of radiation in vacuum: synchrotron radiation, undulator radiation. Examples of radiation due to material boundaries: transition radiation, diffraction radiation, Smith-Purcell radiation, geometric wakefields in accelerators.

Radiation problems in free space are much easier to study than the ones with the material boundaries involved.

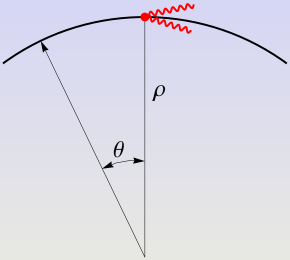
# EM field in free space - a collection of spherical shells

A moving point charge (along an arbitrary trajectory) constantly emits spherical waves of electromagnetic field. [Not all of this field is radiation.] These shells expand with the speed of light from the emission point and fill out the whole space. Red dots are subsequent positions of the charge; they are the centers of the spherical waves. The black point is the current position of the charge.



If the charge is relativistic these waves concentrate in the forward direction.

# Synchrotron radiation of relativistic particles




An observer is located at large distance $r$ from the radiation point in the plane of the orbit. He can measure electric field as a function of time. The bending radius is $\rho$, position on the ring is characterized by $\theta$. Return

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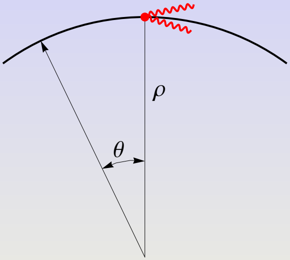


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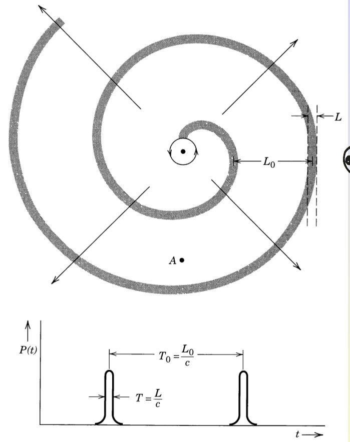
We assume a relativistic particle with charge $q$, velocity $v$,

$$\gamma = (1 - v^2/c^2)^{-1/2} \gg 1.$$

For 1 GeV electron beam $\gamma \approx 2000$, at the LCLS $\gamma \approx 2.8 \times 10^4$.
What are the properties of the synchrotron radiation?

# Synchrotron radiation of relativistic particles

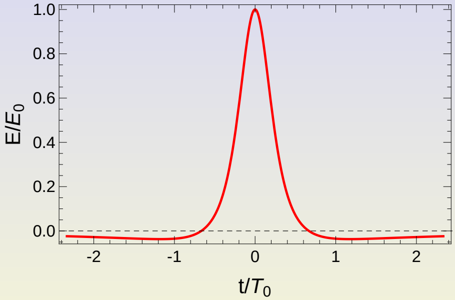



What does the observer see?

From the book "Classical Electrodynamics" by J. Jackson. The shape of the pulsed is not quite right...

# Pulse of synchrotron radiation in the plane of the orbit

Pulse shape in the plane of the orbit (there is a simple [parametric] equation for this curve).




Units of time and field:

$$T_0 = \frac{\rho}{c\gamma^3}, \quad E_0 = \frac{4e\gamma^4}{r\rho}$$

$r$ is the distance from the emission point [radiation is characterized by $E \propto 1/r$]

In textbooks the *critical frequency* is often defined, $\omega_c = \frac{3}{2}\gamma^3 c/\rho$, so the unit of time is $T_0 = (3/2)\omega_c^{-1}$. Tails:

$$E \approx -\frac{1}{6} E_0 \left(\frac{T_0}{|t|}\right)^{4/3}$$

# Characteristics of the synchrotron radiation

- The duration of the core part of the pulse $T_c \sim T_0 \sim \rho/c\gamma^3$. Hence the Fourier harmonics in the spectrum have frequencies $\omega \sim 1/T_c \sim \omega_c$.
- The area under the pulse is zero $\rightarrow$ no radiation at zero frequency. This is a general property of radiation pulses from an emitter of *finite* size. Zero frequency field is static, it drops off as $1/r^2$ and hence is not radiated.

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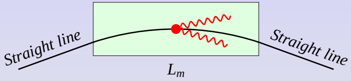
Numerical example for parameters of NSLS-II. Energy 3 GeV, bending magnetic field 0.4 T, bending radius $\rho = 25$ m. The critical frequency $\omega_c = 3.6 \times 10^{18}$ 1/s corresponding to the wavelength $\lambda = 0.5$ nm and the photon energy 2.4 keV.

# Radiation from a short magnet

In practice radiation is generated in dipole magnets of finite length.

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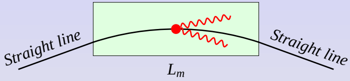


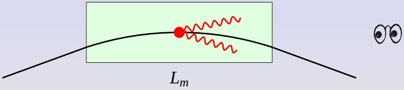

How does the finite length of the magnet $L_m$ affect the properties of the radiation?

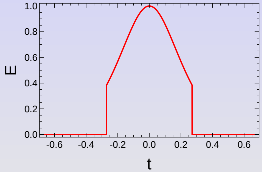
We have to look at the equation of the synchrotron pulse. $E(t)$ is given by a parameteric dependence $E(x)$ and $t(x)$:

$$\frac{E}{E_0} = \frac{1 - x^2}{(1 + x^2)^3}, \quad \frac{t}{T_0} = \frac{1}{2}x + \frac{1}{6}x^3,$$

with $x = \gamma\theta$ fig. Hence each point on the orbit corresponds to a point on the $E(t)$ curve. A simple recipe to draw the pulse generated by a short magnet: draw points corresponding to the part of the circular orbit in the dipole and set $E = 0$ outside of the magnet (on the straight lines).

# Radiation from a short magnet, formation length



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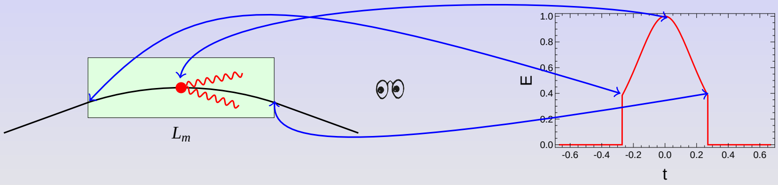

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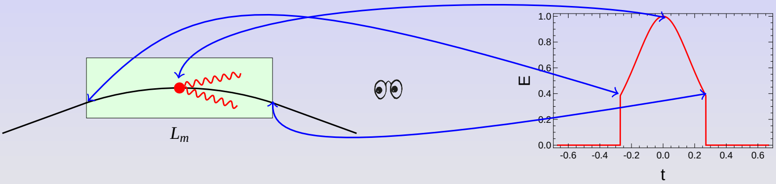


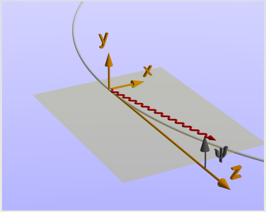

What is the length of the magnet that critically truncates the pulse? It corresponds to the values $x \sim 1 \to \theta\gamma \sim 1$ or

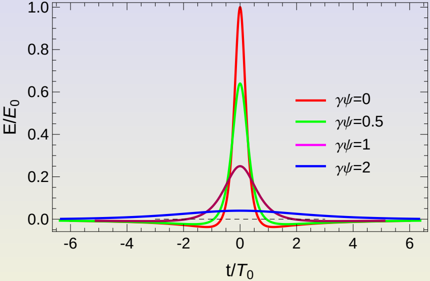
$$L_m \sim \rho\theta \sim \frac{\rho}{\gamma} \sim l_{\|}$$

This is the *longitudinal formation length* of the synchrotron radiation. It is valid for the core of the pulse that carries frequencies $\sim \omega_c$.

# Angular spread of radiation

Plots of EM pulses for $\psi = 0, 0.5/\gamma, 1/\gamma, 2/\gamma$. Polarization: the electric field is in $x - z$ (orbit) plane [σ-polarization].




As ψ increases, the pulses become wider and smaller → radiation is limited to small angles, $\Delta\psi \sim 1/\gamma$.

# What energy is radiated on length $l_{\|}$?

$$W = \frac{c}{4\pi} E^2 \times r\Delta\psi \times r\gamma^{-1} \times T_0$$
$$= \frac{c}{4\pi} \left(\frac{e\gamma^4}{r\rho}\right)^2 \times r\left(\frac{1}{\gamma}\right) \times r\left(\frac{1}{\gamma}\right) \times \frac{\rho}{c\gamma^3} \sim \frac{e^2\gamma^3}{\rho}$$

This is the energy in one pulse which passes through the observation point during time $T_0$ through the area $r\Delta\psi \times r\gamma^{-1}$.

How many photons? Most of the energy is in frequencies $\sim \omega_c$.

$$N_{ph} \sim \frac{W}{\hbar\omega_c} \sim \frac{e^2}{\hbar c} = \alpha \approx \frac{1}{137}$$

Radiation from a magnet of length $l_m$ ($l_m \gg l_{\|}$) is $l_m/l_{\|}$ times larger.

## Spectrum of radiation

An important characteristic of radiation is its spectrum $\mathcal{W}_\omega$:

$$W = \int_0^\infty d\omega \mathcal{W}_\omega$$

Order of magnitude estimate for $\mathcal{W}_\omega$ in the main part of the spectrum, $\mathcal{W}_{\omega,c}$

$$\int_0^\infty d\omega \mathcal{W}_\omega \sim \mathcal{W}_{\omega,c}\omega_c$$

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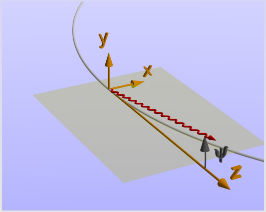
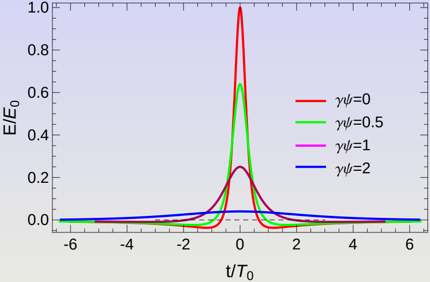
What are characteristics of radiation in the low-frequency part of the spectrum, $\omega/\omega_c \ll 1$? We saw that $\omega_c$ usually corresponds to x-ray range of frequencies. For optical diagnostic of the beam (and in the theory of CSR wakefields) we are interested in wavelength of visible light ($\lambda \sim 1\,\mu$m), that is $\omega/\omega_c \ll 1$.

# Angular spread at small frequencies is larger



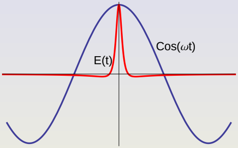

It turns out that when $\psi \gtrsim \gamma^{-1}$ the width of the pulse increases as $\psi^3$, $T \sim T_0(\psi\gamma)^3$ (for $\psi \gtrsim \gamma^{-1}$). Hence the angular spread of radiation of the frequency $\omega \sim 1/T$ is

$$\Delta\psi(\omega) \sim \gamma^{-1} \left(\frac{\omega_c}{\omega}\right)^{1/3} \sim \left(\frac{c}{\rho\omega}\right)^{1/3}$$

# Longitudinal formation length for $\omega \ll \omega_c$

Previously estimated formation length corresponded to $\omega \sim \omega_c$. What is the formation length for $\omega \ll \omega_c$? Take the spectrum of the electric field pulse

$$E_\omega = \int_{-\infty}^{\infty} E(t) e^{i\omega t} dt$$




The frequency $\omega \ll \omega_c$ is formed by the tails of the pulse $t \sim 1/\omega \gg T_0$

$$\frac{t}{T_0} \approx \frac{1}{6} x^3 \sim (\theta\gamma)^3$$

Hence, $\gamma\theta \sim (t/T_0)^{1/3} \sim (\omega_c/\omega)^{1/3}$, or $[\lambda\!\!\!^{-} = \lambda/2\pi = c/\omega]$

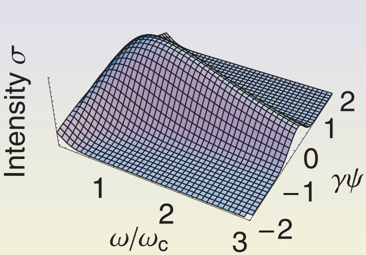
$$l_{\|}(\omega) = \rho\theta \sim \frac{\rho}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/3} \sim \rho\left(\frac{c}{\rho\omega}\right)^{1/3} \sim (\lambda\!\!\!^{-}\rho^2)^{1/3},$$

$l_{\|}(\omega)$ does not depend on $\gamma$. Note that

$$l_{\|}(\omega) \sim \lambda\!\!\!^{-}/\Delta\psi(\omega)^2$$

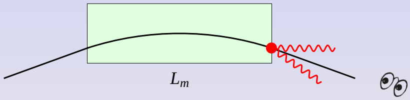
# Spectrum of the radiation versus ω and ψ

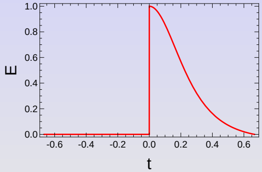
Spectral intensity of the synchrotron radiation for two polarizations of the electric field (σ—in the orbit plane, π—perpendicular to the plane)




We now how an understanding of the width of these plots along ψ and an estimate of the height.

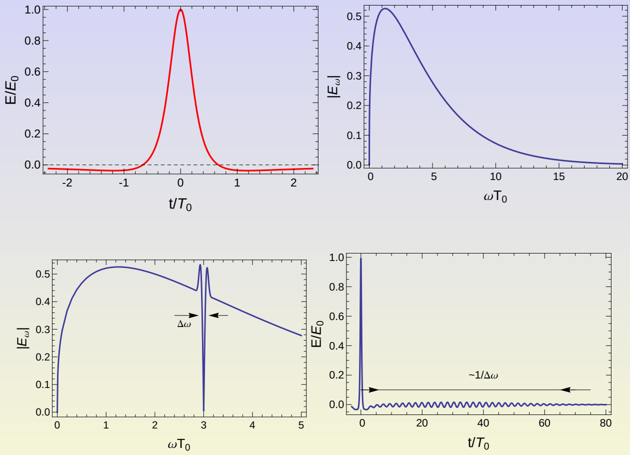
# Edge radiation




*Edge* radiation is generated at the exit from the magnet (assuming a hard edge magnetic field distribution in the magnet). Since for a discontinuity $E_\omega \propto 1/\omega$ the edge radiation is more intensive in the range of low frequencies. Edge radiation can be used as a source of infrared radiation (R. Bosch, Il Nuovo Cimento, 1998).

# A hole in spectrum—example of spectral manipulation



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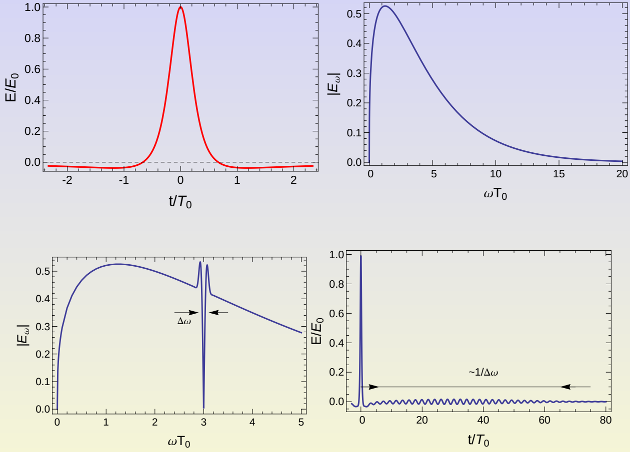
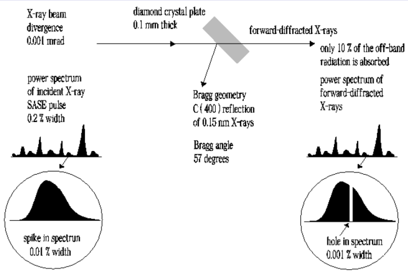
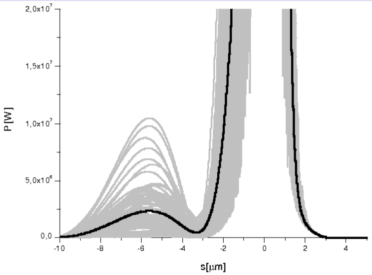



Sending radiation pulse through a thin crystal can remove a small part of the spectrum.

# Hard x-ray self seeding

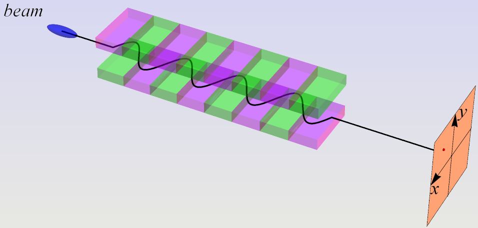
G. Geloni, V. Kocharyan and E. Salding, DESY 10-133 (2010).




Experiment on soft x-ray seeding is being prepared at LCLS.

# Undulator radiation



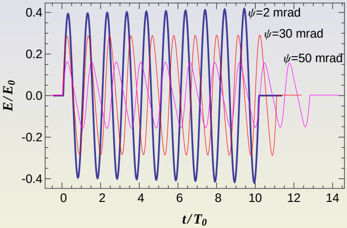

Undulator period $\lambda_u$, $k_u = 2\pi/\lambda_u$. The undulator parameter K

$$K = \frac{eB}{k_u mc}$$

has a meaning of the ratio of max wiggling angle of the orbit to the parameter $\gamma^{-1}$.

# Undulator radiation, $K \ll 1$

Electric field $E(t)$ for a weak undulator, $K = 0.1$ (top), at large distance from the exit of the undulator, in the plane of oscillations ($y = 0$), at $\psi = 2$, 30 and 50 mrad angle relative to the axis ($\psi = x/z$). The undulator has 10 periods and $\gamma = 10$.




The frequency of the radiation decreases with angle $\psi$. For $K \ll 1$ the field has sinusoidal form $\rightarrow$ the spectrum of $E(t)$ has a narrow peak around $\omega_{\mathrm{rad}}(\psi)$ and the width of the spectrum $\Delta\omega/\omega \sim 1/N_u \ll 1$ if $N_u \gg 1$.

From theory:

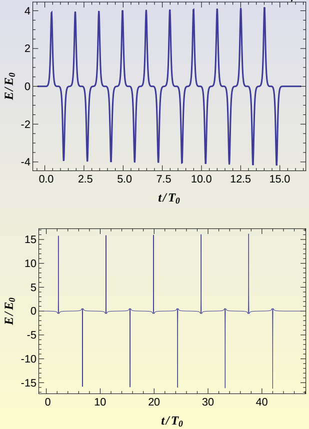
$$\lambda_{\mathrm{rad}}(\psi) = \frac{\lambda_u}{2\gamma^2}(1 + \gamma^2\psi^2)$$

[In the figure $T_0 = \pi/\gamma^2 k_u c$.]

# Undulator radiation, $K \gtrsim 1$

$$\lambda_{\rm rad}(\psi) = \frac{\lambda_u}{2\gamma^2 h}(1 + \gamma^2\psi^2 + K^2/2)$$

($h$ is the harmonic number, $h = 1, 2, 3\ldots$).




Electric field $E(t)$ for 2 different undulator parameters, $K = 1$ (top) and 4 (bottom) at large distance from the exit of the undulator (the same undulator periods and $\gamma$ as on the previous slide). Note increasing duration of the pulse with $K$ (in accordance with the formula). For $K \gg 1$ each spike is a coherent radiation pulse $\to$ much broader spectrum of the radiation $\to$ large values of $h$.

# Undulator radiation, formation length

What is the formation length $l_\parallel$ for the undulator radiation? Each wiggle is emitted by one period of the undulator,

$$l_\parallel \sim L_u$$

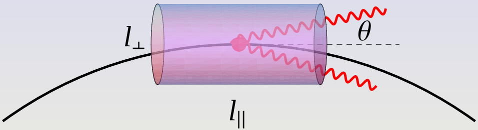
What is the angular spread of the undulator radiation? Use the formula that relates the angular spread with the formation length ($l_\parallel(\omega) \sim \lambda/\Delta\psi(\omega)^2$)

$$\Delta\psi(\omega) \sim \sqrt{\frac{\lambda}{l_\parallel}} \sim \sqrt{\frac{\lambda}{L_u}}$$

This is called the central cone radiation opening angle.

# Transverse formation length for synchrotron radiation

In addition to the longitudinal extension, a particle needs some transverse space $l_\perp$, the *transverse formation length* or *transverse coherence size*, to be able to form radiation.

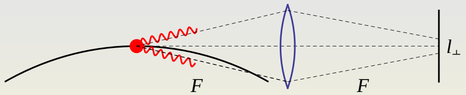



How to estimate $l_\perp$? If the angular spread of radiation is $\theta$ ($\theta \ll 1$), then the transverse wavenumber is $k_\perp = k \sin\theta \sim k\theta$. The angle $\theta(\omega) \sim \Delta\psi(\omega)$ for the radiation coming from the formation length.

$$l_\perp(\omega) \sim \frac{1}{k_\perp} \sim \frac{1}{k\theta(\omega)} \sim \frac{\lambda}{\theta(\omega)} \sim \frac{\lambda}{(c/\rho\omega)^{1/3}} \sim \lambda^{2/3}\rho^{1/3}$$

# Transverse formation length

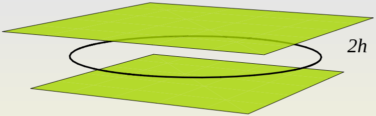
There are several important properties of the transverse formation length: *a)* image of the source of radiation, *b)* shielding of the radiation, *c)* coherence effects.
Make an image of a radiated electron with 1:1 optics. Due to the small angular spread the diffraction will smear out the image.

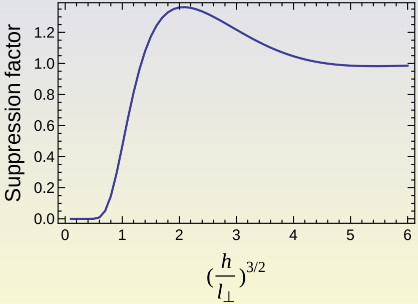



The the spot size on the screen is of order of $l_\perp$. While an electron has zero dimensions (in classical electromagnetism), using its radiation, one can only localize it transversely within the area $\sim l_\perp^2$.

# Shielding by metallic walls

If metallic walls are placed close to the beam, within the transverse formation length, they suppress the radiation—wall shielding effect. Here is the suppression factor for two parallel plates for a given frequency (at $\omega \ll \omega_c$).

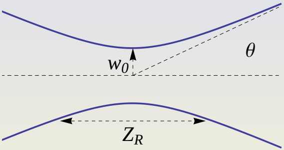


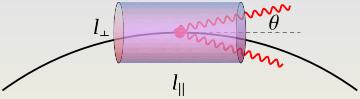




Reference: J. Murphy, S. Krinsky and R. Gluckstern, Particle Accelerators, 1997.

# Analogy between a laser beam and radiation

A Gaussian laser beam near focus is characterized by the waist $w_0$ and the Rayleigh length (the depth of the focus) $Z_R$ that are related to the convergence angle $\theta$. Formation lengths $l_\perp$ and $l_\parallel$ are their respective analogs.




$$w_0 \sim \frac{\lambda}{\theta}$$

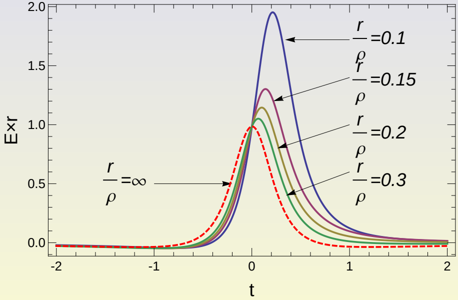
$$Z_R \sim \frac{\lambda}{\theta^2}$$

$$l_\perp \sim \frac{\lambda}{\theta}$$

$$l_\parallel \sim \frac{\lambda}{\theta^2}$$

# How far is the far zone?

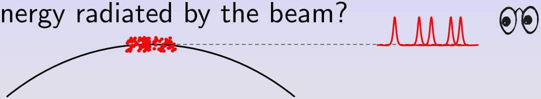
How far from the radiation point an observer should be located in order to see the radiation field (far zone)? Plot of a synchrotron pulse for 4 different distances $r$ from the radiation point $r/\rho = 0.1,\ 0.15,\ 0.2,\ 0.3,\ 2.$ $\gamma = 10,\ \rightarrow l_{\|} \sim 0.1\rho$. Polarization is with electric field in plane.




Conclusion: the observer should be $\gg l_{\|}$ from the radiation point.

# Coherent versus incoherent radiation

A bunch has many particles, and each radiates its own pulse.
What is the energy radiated by the beam?



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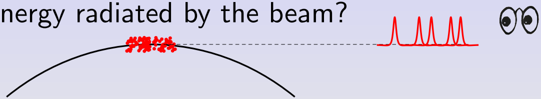

If the pulses do not overlap, the energy is the sum of the energies radiated by each electron. Let's estimate the condition of non-overlapping for a relativistic beam with (peak) current $I$. The distance between electrons

$$d \sim \frac{ec}{I} \sim r_e \frac{I_A}{I}$$

$I_A = mc^3/e = 17$ kA - Alfven current, $r_e = 2.8 \times 10^{-13}$ cm. For the current 1 A $d \sim 0.05$ nm.

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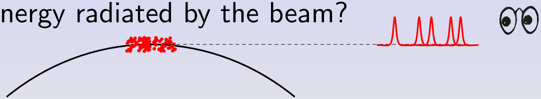

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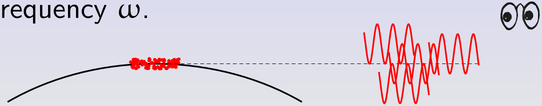
$I_A = mc^3/e = 17$ kA - Alfven current, $r_e = 2.8 \times 10^{-13}$ cm. For the current 1 A $d \sim 0.05$ nm.
This analysis is too restrictive (and somewhat flawed).

# Coherence: Fourier analysis

We need to analyze coherence in terms of Fourier harmonics of the pulses. Decompose each pulse into Fourier integral and consider only a given frequency $\omega$.



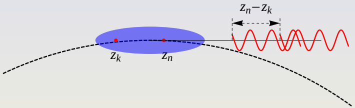
$$E_n(t) \to \hat{E}_\omega e^{i\omega t + i\phi_n}, \qquad \hat{E}_{total}(\omega) = \sum_{n=1}^{N} \hat{E}_\omega e^{i\omega t + i\phi_n}$$

the phase is determined by the time of emission $\to$ longitudinal position of the electron in the bunch. Intensity of radiation $\sim |\hat{E}_{total}|^2$

$$\left| \sum_{n=1}^{N} \hat{E}_\omega e^{i\omega t + i\phi_n} \right|^2 = |\hat{E}_\omega|^2 \sum_{n,j} e^{i(\phi_n - \phi_k)} = |\hat{E}_\omega|^2 \left( N + \sum_{n \neq j} e^{i(\phi_n - \phi_k)} \right)$$

# Incoherent and coherent radiation

The first term in $N + \sum_{n \neq j} e^{i(\phi_n - \phi_k)}$ is the *incoherent* radiation: its intensity is equal to the intensity of one particle $\times N$. The second one is the *coherent* radiation, it can be as large as $N^2$.




In 1D model, $\phi_n = kz_n$ and the sum becomes $\sum_{n \neq j} e^{ik(z_n - z_k)}$.

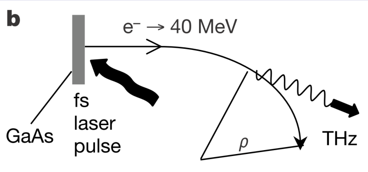
The sum $\sum_{n \neq j} e^{ik(z_n - z_k)} \approx 0$ if two conditions are satisfied

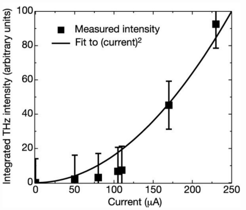
- The bunch is much longer than $\lambda$ (then for a typical electron $kz_n \gg \pi$)
- Positions of different particles are not correlated

For electron bunches in circular machines these two conditions are usually satisfied.

# Incoherent and coherent radiation

One way to generate coherent radiation is to prepare a bunch that is shorter than $\lambda$.




Coherent radiation of a short bunch at NSLS: G. Carr et al. Nature, 2002. Bunch length 0.5 ps (0.15 mm) radiates 0.6 Thz ($\lambda = 0.5$ mm). Measured intensity of THz radiation scaled with the number of particle as $N^2$.

This is not possible for wavelengths of order of nanometers.

# Coherence in a long bunch

In the case $\sigma_z \gg \lambda$ coherent radiation can be obtained by introducing correlations between positions of the particles—by modulating the beam density.

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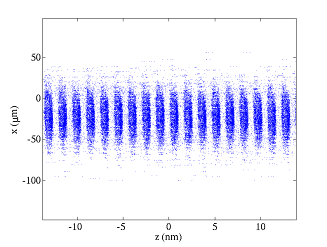
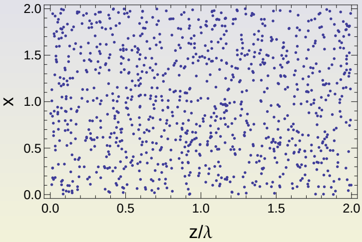
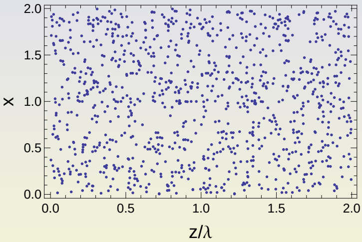



Simulated bunching in the LCLS beam (courtesy of Y. Ding). Particles' positions along $z$ are now correlated.

# Correlations of particle positions in the beam

Coherent effects can amplify the radiation, but they also can suppress it. The beam with distribution on the left graph radiates incoherently, and the beam with the distribution on the right does not radiate at all at the wavelength $\lambda$.

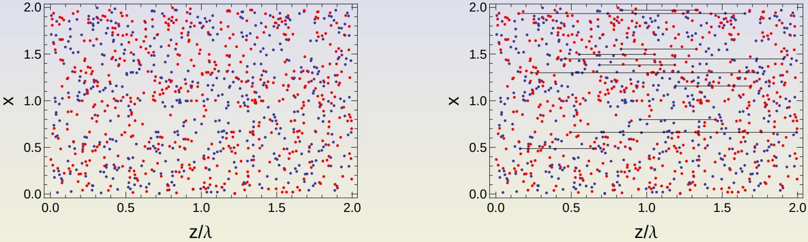



The explanation is on the next slide.

# Correlations of particle positions in the beam

The blue particles are distributed randomly but positions of each red one is shifted by $n\lambda/2$ relative to the blue one with $n$ being an odd number.



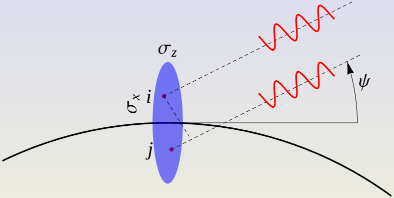

This is used for a quiet start in FEL simulation codes. The idea of noise suppresion in a beam is studied in A. Gover and E. Dyunin, PRL, 2009.

# Effect of transverse size of the beam on coherence

Consider a short bunch ($\sigma_z \leq \lambda$) that radiates coherently sychrotron radiation. If its transverse size becomes larger than $l_\perp$, the coherent radiation becomes suppressed.

Consider the phase difference between particles $i$ and $j$

$$\Delta\phi \sim k\sigma_x \sin\psi \sim k\sigma_x\psi$$




One cannot neglect the transverse size of the beam $\sigma_x$ if $\Delta\phi \gtrsim \pi$, or

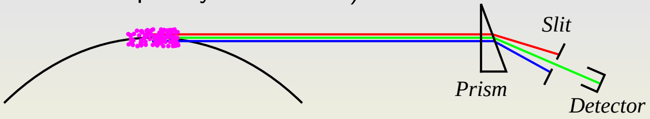
$$\sigma_x \gtrsim \frac{\lambda}{\Delta\psi(\omega)} \sim l_\perp$$

The coherent radiation in the forward direction does not change, so the angular spread becomes smaller.

# Fluctuations of radiation field

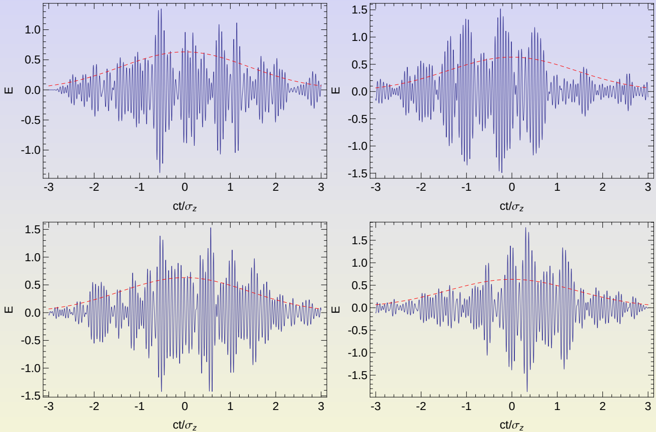
$$\hat{E}_{total}(\omega) = \hat{E}_\omega e^{i\omega t} \sum_{n=1}^{N} e^{ikz_n}$$

Depending on random distribution of particle positions, the intensity of radiation at a given $\omega$ will fluctuate from shot to shot (also from one frequency to another).



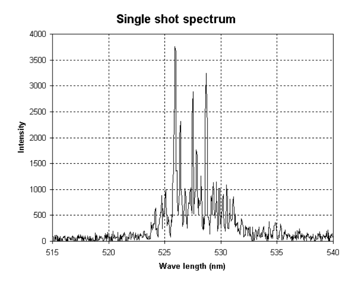

What is $E(t)$ on the detector? It depends on the frequency of the spectral line $\omega_0$, the width of the slit $\Delta\omega$, and the bunch length $\sigma_z$. Next slide: simulations for the case $\omega_0 = 10c/\sigma_z$ and $\Delta\omega = 0.1c/\sigma_z$.

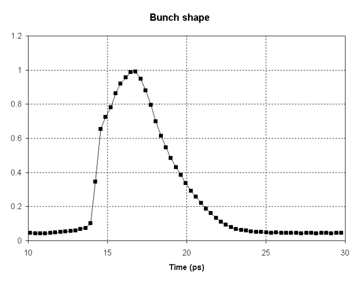
# Fluctuating field of a Gaussian bunch




One can see about 10 coherent regions in the field pattern [$10 \approx c/\sigma_z\Delta\omega$]. Measuring fluctuational properties of the radiation one can infer the beam profile.

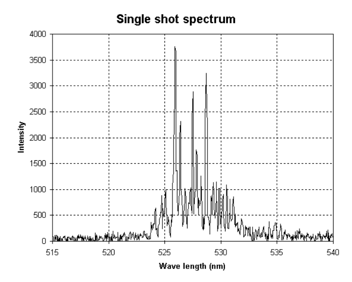
# Experimental measurement of bunch shape from fluctuations

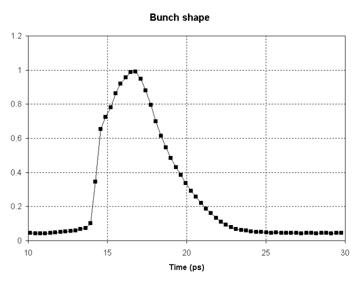



Measured fluctuating spectrum of synchrotron radiation at APS and inferred bunch shape, (V. Sajaev, EPAC2000).

# Experimental measurement of bunch shape from fluctuations




Measured fluctuating spectrum of synchrotron radiation at APS and inferred bunch shape, (V. Sajaev, EPAC2000). Fluctuations also occur in SASE FEL, however they are more complicated, especially in the saturation. See invited talk by M. Yurkov at FEL 2010. Fluctuations of a SASE FEL can be used for measuring the FEL pulse length

# Radiative reaction force

When a bunch is radiating electromagnetic field, the energy is taken from the kinetic energy of the particles. The force that slows down the beam is called the *radiative reaction* force.
For a point charge moving with a non-relativistic velocity

$$\mathbf{F}_{\mathrm{rr}} = \frac{2}{3}\frac{e^2}{c^3}\ddot{\mathbf{v}}$$

$\mathbf{F}_{\mathrm{rr}}$ can be generalized for a relativistic case.

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$\mathbf{F}_{\mathrm{rr}}$ can be generalized for a relativistic case.

There is a trouble with this force: the equation $m\dot{\mathbf{v}} = \mathbf{F}_{\mathrm{rr}}$ has spurious solutions $\sim e^{t/t_0}$ with $t_0 = (2/3)r_e/c$. This, however is not a real issue: use $m\dot{\mathbf{v}} = \mathbf{F}_{\mathrm{rr}} + \mathbf{F}_{\mathrm{ext}}$.

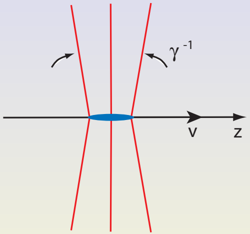
# Radiative reaction forces

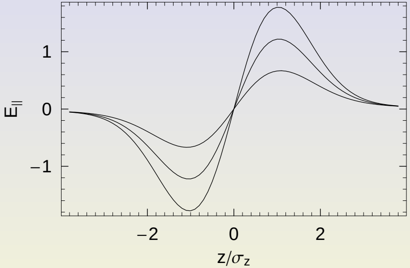
In the process of incoherent radiation, each electron experiences a drag. For coherent radiation, the whole bunch, or a part of it (containing many electrons), experiences collective radiative force. Since coherent radiation can be N times stronger than the incoherent one the collective force can be much larger than the incoherent one. During last decate a lot of effort went to studying this *collective radiative reaction* force (also called the *CSR wakefield*).

A bunch of particle experiences collective electromagnetic forces even when it does not radiate—the *space charge force*.

# Space charge force

A bunch Q with $\sigma_z$ and $\sigma_\perp$ is traveling along a straight line along $z$, with relativistic factor $\gamma$. What are the *longitudinal* forces in this case?




$$E_{sc} \sim \frac{Q}{\sigma_z^2 \gamma^2} \log \frac{\sigma_z \gamma}{\sigma_\perp}$$

This field does not change the total energy of the beam, but redistributes it between different parts of the bunch.

# Intensity of coherent radiation of bunch of length $\sigma_z$

The bunch of length $\sigma_z$ radiates coherently wavelength $\lambda \sim \sigma_z$. We can estimate the coherent radiation of the bunch of length $\sigma_z$: it radiates $\alpha N^2$ photons of frequency $\omega \sim c/\sigma_z$ per formation length $l_{\|}(\omega) \sim (\rho/\gamma)(\omega_c/\omega)^{1/3}$. The energy loss per unit length is the coherent radiation reaction force $F_{rr}$

$$F_{rr} \sim N^2 \frac{\alpha \hbar \omega}{l_{\|}(\omega)} \sim \frac{Q^2}{\sigma_z^{4/3} \rho^{2/3}}$$

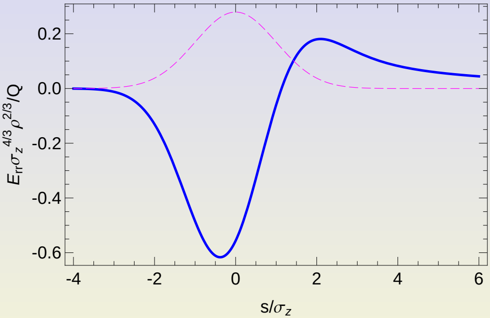
The radiation reaction electric field $E_{\|} = F_{rr}/Q$ hence is

$$E_{\|} \sim \frac{Q}{\sigma_z^{4/3} \rho^{2/3}}$$

This does not depend on $\gamma$. For $Q = 1$ nC, $\sigma_z = 50$ µm, $\rho = 10$ m, $E_{\|} \sim 1$ MV/m. The Coulomb field for this example ($\gamma = 10^3$) $E_{sc} \sim 25$ kV/m.

# CSR wake

The trouble is that $E_{\|}$ varies inside the bunch. Plot of the steady state CSR wake for a Gaussian bunch.




The head is accelerated and the tail slows down $\rightarrow$ they move along different trajectories in the dipole $\rightarrow$ transverse emittance growth.

# Formation length and the CSR wake

Since formation lengths are crucial for radiation, they are important for the radiative force as well.

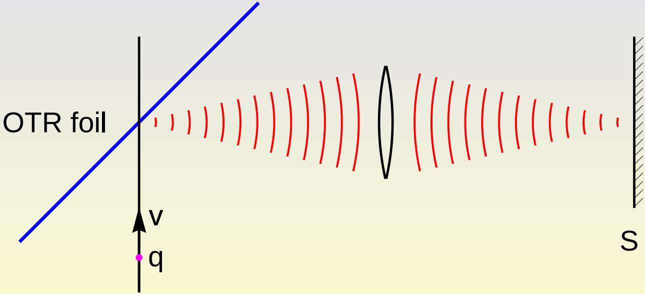
An often used model of the CSR wake is that of an infinitely thin (pencil-like) beam , $\sigma_x, \sigma_y \to 0$, moving on a circle. When this model is valid? Answer: a) when the transverse size, $\sigma_x$, $\sigma_y$ of the beam is smaller than the transverse formation length $l_\perp$, b) the length of the magnet $l_m$ is longer than the longitudinal formation length, $l_m \gg l_\parallel$.
Estimates: $\sigma_z = 100\ \mu$m, $\rho = 10$ m,

$$l_\perp \sim \lambda^{2/3}\rho^{1/3} \sim \sigma_z^{2/3}\rho^{1/3} \approx 4.5\ \mathrm{mm}$$

The "pencil" beam modes is valid even if $\sigma_\parallel \gg \sigma_z$!
References: Ya. Derbenev et al. TESLA FEL-Report 1995-05; J. Murphy, S. Krinsky and R. Gluckstern, Particle Accelerators, 1997.

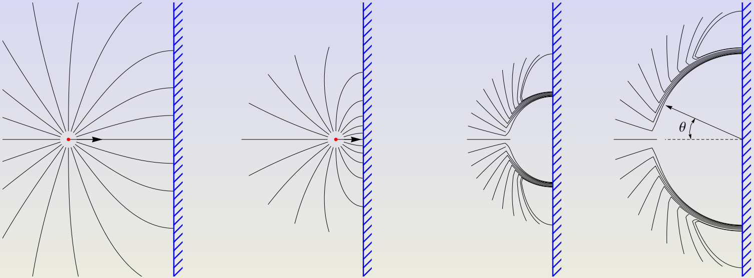
# Transition radiation

Transition radiation occurs when a particle crosses a boundary of two media with different electrodynamic properties. We will consider a boundary between vacuum and metal. Applications in FELs - OTR (optical transition radiation) screens for beam diagnostics.



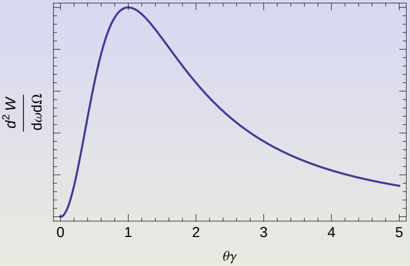
# Transition radiation (normal incidence)

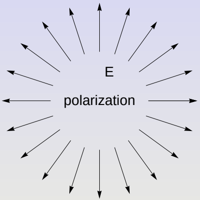
A point charge hitting metallic foil ($beta = 0.7$).




After the charge hits the metal it disappears, and an expanding spherical wave annihilates the electromagnetic field moving with the charge. Radiation field is localized in a thin spherical shell. For a relativistic particle, $\gamma \gg 1$, it is concentrated at small angles, $\theta \sim 1/\gamma$.

# Angular spread and polarization




Angular distribution of the spectral intensity of the transition radiation (assuming $\theta \ll 1$) is

$$\frac{d^2\mathcal{W}}{d\omega d\Omega} \approx \frac{q^2}{c\pi^2} \frac{\theta^2}{(\gamma^{-2} + \theta^2)^2}$$

# Transition radiation

The angular spread $\Delta\theta \sim 1/\gamma$. What are the transverse and longitudinal formation lengths?

$$l_{\parallel} \sim \frac{\lambda}{\Delta\theta^2} \sim \lambda\gamma^2, \qquad l_{\perp} \sim \frac{\lambda}{\Delta\theta} \sim \lambda\gamma$$

For $\gamma \sim 10^3$ and $\lambda \sim 0.1\ \mu\text{m} \to l_{\parallel} \sim 0.1\ \text{m}$, $l_{\perp} \sim 100\ \mu\text{m}$. What is the meaning of these numbers?

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The transverse size of the foil should be larger than $l_{\perp}$—not a problem for visible radiation.

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The transverse size of the foil should be larger than $l_{\perp}$—not a problem for visible radiation.

The size of the image of one electron is $\sim l_{\perp}$—very poor resolution?

# Transition radiation

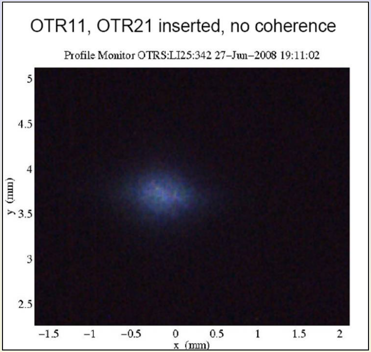
We have to revise the estimate for $\Delta\theta$. While the spectrum decay with $\theta$, there are a still plenty of energy at angles $\gg \gamma^{-1}$:

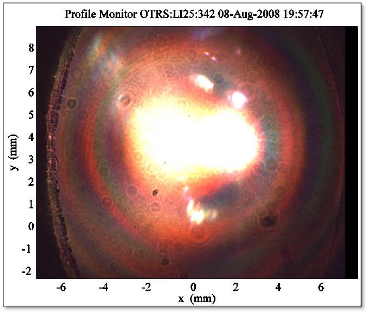
$$\int d\Omega \frac{d^2\mathcal{W}}{d\omega d\Omega} = 2\pi \int_0^{\theta_{\max}} \theta d\theta \frac{q^2}{c\pi^2} \frac{\theta^2}{(\gamma^{-2}+\theta^2)^2} \approx \frac{2q^2}{c\pi} \ln(\gamma\theta_{\max})$$

A better estimate for $\Delta\theta \sim \theta_{\max}$, that is the collection angle of the diagnostic. For a LCLS OTR screen $\theta_{\max} \sim 75$ mrad. This makes $l_\perp \sim$ $\mu$ms. It also changes the longitudinal formation length.

# Transition radiation

Pictures of OTR and COTR from the LCLS.



# Summary

- Order of magnitude estimates are an essential element of understading of a physics phenomenon.
- Important parts of radiation processes are: the pulse shape, the spectrum, the angular distribution, the longitudinal and transverse formation lengths. There are universal relations between the angular spread and the formation lengths.
- Coherent radiation is due to correlations between position of particles in the bunch. Randomness of these position leads to fluctuation processes in radiation.
- Radiative reaction forces keep the energy balance in the process of radiation (both incoherent and coherent)
- Transition radiation—simplest example of radiation processes in the presence of material boundaries.