MaRIE

(Matter-Radiation Interactions in Extremes)

Three-Dimensional Coherent Synchrotron Radiation Calculations

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Overview

- Coherent synchrotron radiation (CSR) and its effect on beam emittance growth
- One-dimensional CSR approaches and limitations
- Three-dimensional CSR calculations and comparison with the 1-D model
  - Exact Lienard-Weichert solver
  - CSRTrack
  - Finite-difference time-domain (FDTD) solver
  - Comparison of longitudinal and transverse force amplitudes
  - 100 MeV and 1 GeV
- CSR enhancement from a micro-bunched beam

MaRIE builds on the LANSCE facility to provide unique experimental tools to meet this need

First x-ray scattering capability at high energy and high repetition frequency with simultaneous charged particle dynamic imaging

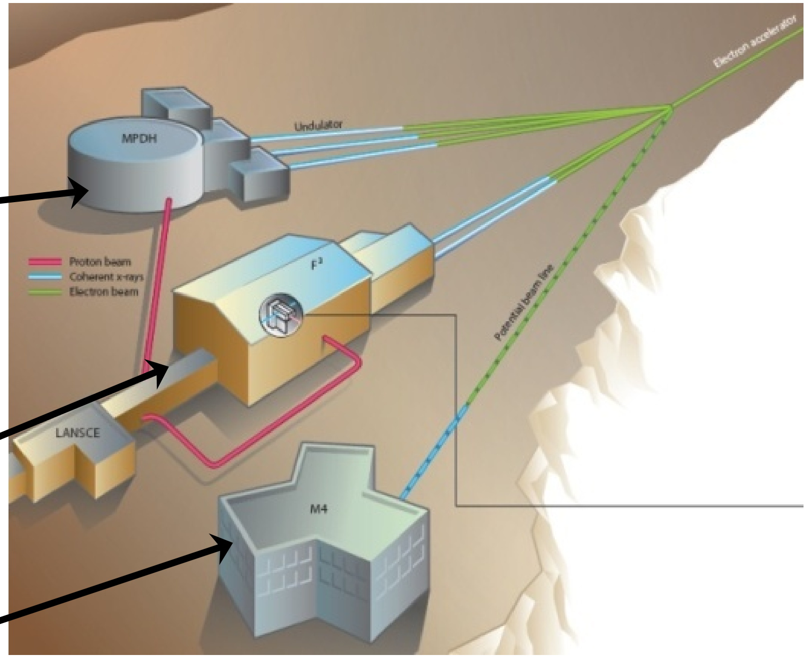
(**MPDH**: Multi-Probe Diagnostic Hall)

Unique in-situ diagnostics and irradiation environments beyond best planned facilities

(**F^3**: Fission and Fusion Materials Facility)

Comprehensive, integrated resource for materials synthesis and control, with national security infrastructure

(**M4**: Making, Measuring & Modeling Materials Facility)



- **Accelerator Systems**
  - Electron Linac w/XFEL
  - LANSCE proton accelerator power upgrade
- **Experimental Facilities**
- **Conventional Facilities**

MaRIE will provide unprecedented international user resources

XFEL Requires Tiny Emittances

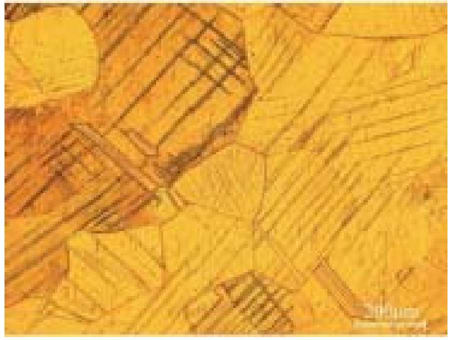
Beam energy is typically chosen because of two constraints:

$$\frac{\varepsilon_{beam}}{\gamma} = \varepsilon_{lab} \leq \frac{\lambda_{x-ray}}{4} = \frac{\lambda_{wiggler}}{8\gamma^2}\left(K^2+1\right)$$

The choice for beam energy (γ) is dominated by the beam emittance, not wiggler period (which can go down to 1 cm)

Energy diffusion limits how high the beam energy can be (~ 20 GeV), puts a very extreme condition on the beam emittance (ideally ~ 0.15 μm)

$$\left\langle \frac{d}{dz}\left(\delta\mathcal{E}\right)^2_{QF} \right\rangle = \frac{55e\hbar\gamma^4 r_e^2 B_w^3}{24\sqrt{3}m_e c}$$

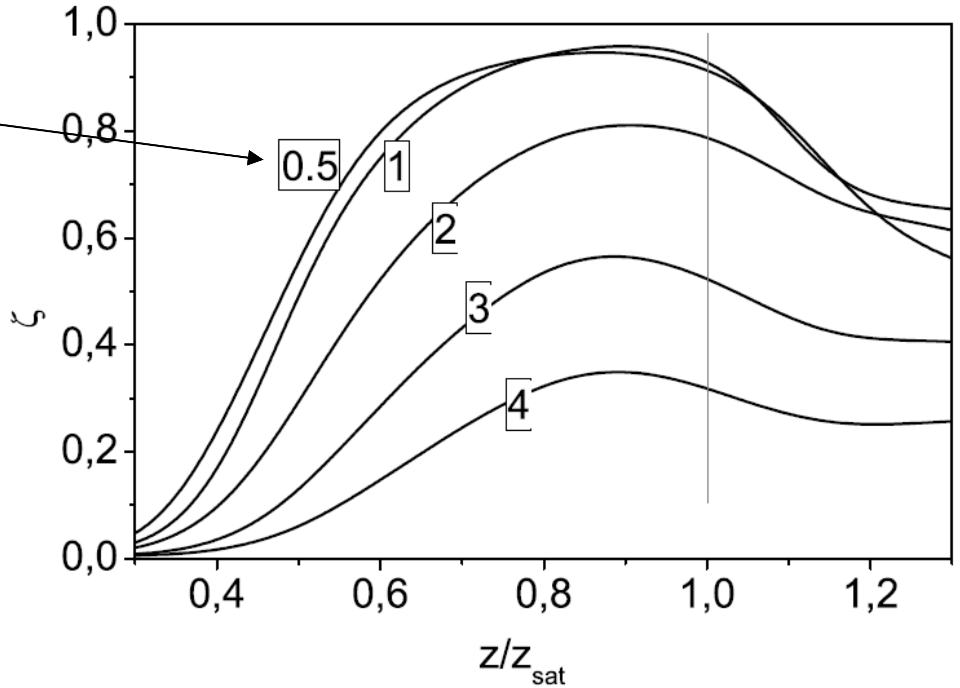


Typical 100-μm sized grain boundaries we want to resolve

Transverse Coherency Also Requires Tiny Emittances

$$\hat{\varepsilon} = 2\pi\varepsilon_n / \gamma\lambda_{X-ray}$$

$$\zeta = \frac{\iint \left|\Gamma\left(r_\perp r'_\perp\right)\right|^2 \left\langle I(r_\perp) I(r'_\perp)\right\rangle dr_\perp dr'_\perp}{\left[\int \left\langle I(r_\perp)\right\rangle dr_\perp\right]^2}$$



An emittance of 0.15 μm is an emittance ratio of about 1 for the figure above (at 20 GeV). MaRIE XFEL baseline emittance (0.3 μm) leads to a transverse coherency of about 0.8.

We're Working in a New Emittance and Bunch Length Regime

Emittance = 0.15 μm
Charge = 50 pC
Current = 3.4 kA (15-fsec, 5-μm long)

Additionally, we might like to have a pre-microbunched electron beam for seeding the XFEL

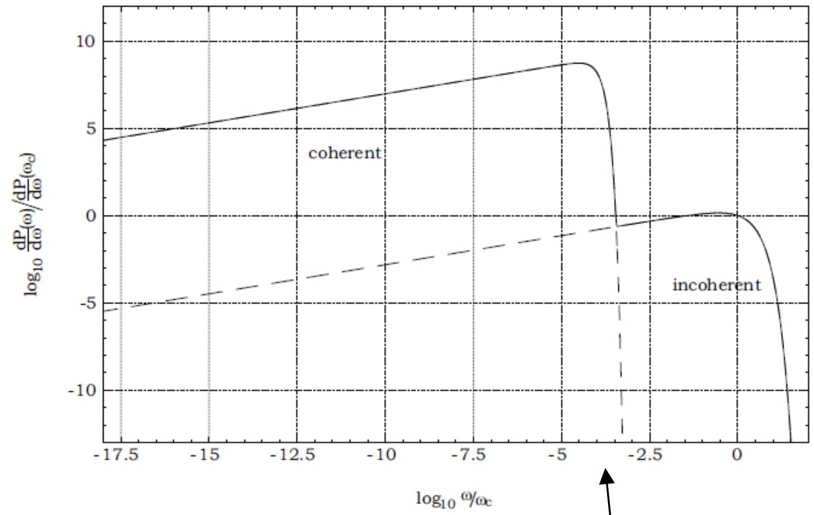
These constraints motivate us to have a more refined understanding of CSR effects, especially for very short electron bunches and for electron bunches with a microbunched current structure.

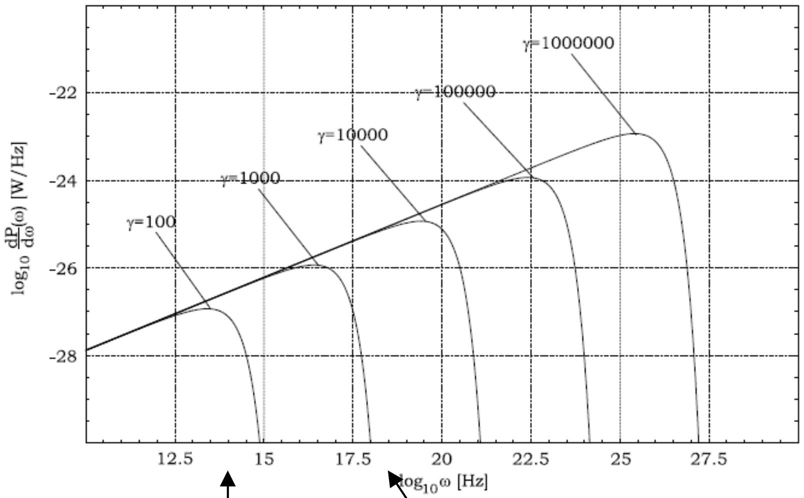
Here we report of a new numerical study of CSR using an exact Lienard-Wiechert solver using the full-bunch number of electrons.

We're Moving Out Of The Regime The 1-D Representations Are Known To Work

Ideal ISR/CSR plot for 1-D CSR representation

Small or nonuniform bunches can lead to a different scaling





Plots from Stulle's Thesis

$$\omega_{bunch} = \frac{2\pi c}{\sigma_z}$$

$\sigma_z = 10\ \mu\text{m}$ $\qquad$ $\sigma_z = 1\ \text{nm}$

CSR Force – Gaussian Bunch

Numerical representations tend to be one-dimensional (CSRTrack is a notable exception).

The idea is all transverse positions are collapsed onto a central orbit and a 1-D charge distribution drives a Green's function approach.

Exact CSR-induced field for a ultra-relativisitic 1-D Gaussian beam (Goldreich *Astro. Journ.* 1971):

$$E_\theta = -\frac{1}{4\pi\varepsilon_0}\frac{\Gamma(2/3)}{3^{1/3}}\sqrt{\frac{2}{\pi}}\frac{eN_0}{a^2\sigma^{4/3}}e^{-\theta^2/4\sigma^2}D_{1/3}\left(\frac{\theta}{\sigma}\right)$$

$$N(\theta) = \frac{N_0}{\sigma}\frac{1}{\sqrt{2\pi}}e^{-\theta^2/2\sigma^2} \qquad \int_{-\infty}^{\infty} N(\theta)\,d\theta = N_0 \qquad \int_{-\infty}^{\infty}\theta^2 N(\theta)\,d\theta = N_0\sigma^2$$

Note that $e^{-\theta^2/4\sigma^2}D_{1/3}\left(\frac{\theta}{\sigma}\right)$ has a maximum of 0.823 when $\theta/\sigma = 0.382$

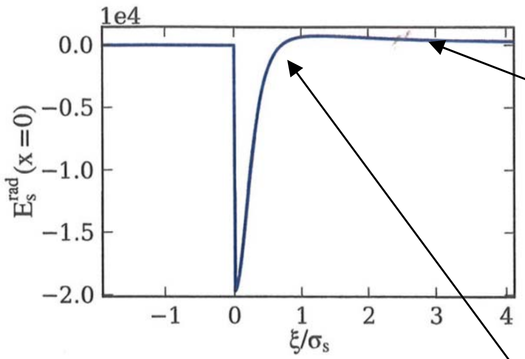
Single-Particle Wake Function

from M,K,G 1995 PAC

$$4\pi\varepsilon_0 E_\theta = 0 \qquad \mu < 0 \qquad \left(\mu = \frac{3}{2}\gamma^3 \frac{s}{R}\right)$$

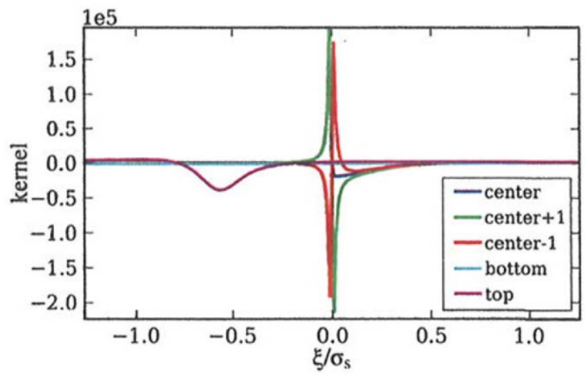
$$= -\frac{4e\gamma^4}{3R^2}\frac{1}{2} \qquad \mu = 0$$

$$= -\frac{4e\gamma^4}{3R^2}\frac{d}{d\mu}\left\{\frac{9}{4}\frac{\cosh\left[\frac{5}{3}\sinh^{-1}\mu\right] - \cosh\left[\sinh^{-1}\mu\right]}{\sinh\left[2\cosh^{-1}\mu\right]}\right\} \qquad \mu > 0$$



CSR wake is mostly from this stuff out here

$$s_{width} = \frac{R}{3\gamma^3}$$



Longitudinal field including off nominal trajectory

1-D CSR Field Representations

Numerical representations for arbitrary distributions:

Same for ultra-relativistic electrons

Saldin formula: $$\Delta E_\theta = \frac{d\lambda}{dz}\frac{2}{4\pi\varepsilon_0 3^{1/3} s^{1/3} \rho^{2/3}}\Delta s$$

LANL formula: $$\Delta E_\theta = \frac{\Delta\lambda}{4\pi\varepsilon}\left[\frac{1}{r_{ret} - \vec{r}_{ret}\cdot\vec{u}_{ret}/c}\left(\frac{1}{\gamma^2} - \beta^2\frac{x}{R} + \beta^2\frac{r}{R}(1-\cos\zeta_{ret})\right)\right]_{\zeta_i}^{\zeta_f}$$

1-D retarded time equation: $$2\beta\sin\frac{\zeta_{ret}}{2} = \zeta_{ret} - \zeta$$

$$\zeta_{ret} = (24\zeta)^{1/3}$$ ultra-relativistic

need to use if energy lower than $$\gamma < \frac{R^{1/3}\sqrt{12}}{\delta^{1/3} 24^{1/3}}$$

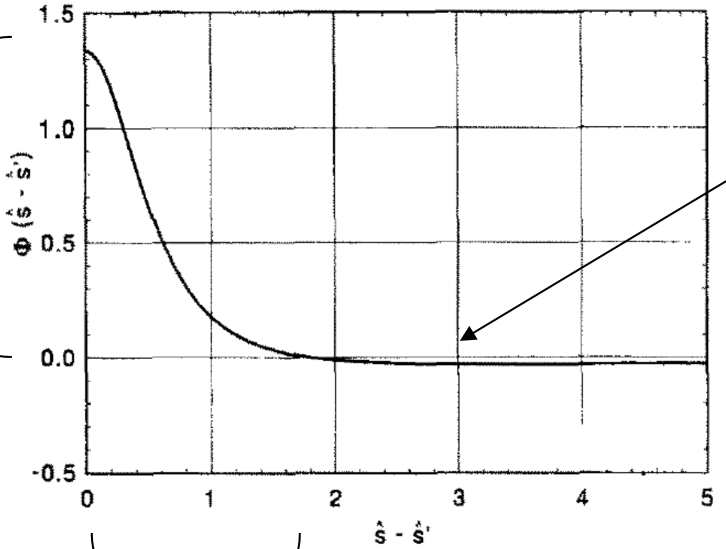
(100 MeV for a 10-micron bunch)

$$\zeta_{ret} = \left(s^3\right)^{1/3} - \left(t^3\right)^{1/3}$$ non ultra-relativistic

$$s^3 = -\frac{B}{2} + \sqrt{\frac{B^2}{4} + \frac{B^3}{27}} \quad t^3 = \frac{B}{2} + \sqrt{\frac{B^2}{4} + \frac{B^3}{27}} \quad A = \frac{24}{\beta}(1-\beta) \quad B = -\frac{24}{\beta}$$

1-D Form Comes From The Single-Particle Wake Function

$\sim \gamma^4$



$\frac{R}{3\gamma^3}$

$$\Phi = K(s-s')$$

$$= \frac{2e^2}{3^{3/2} R^{2/3} (s-s')^{4/3}} \qquad \text{for } (s-s') >> \frac{R}{\gamma^3}$$

$$Wake = \int_{-\infty}^{s} \lambda(s') K(s-s') ds'$$

$$= -\frac{2e^2}{3^{1/3} R^{2/3}} \int_{-\infty}^{s} \frac{1}{(s-s')^{1/3}} \frac{d\lambda(s')}{ds'} ds'$$

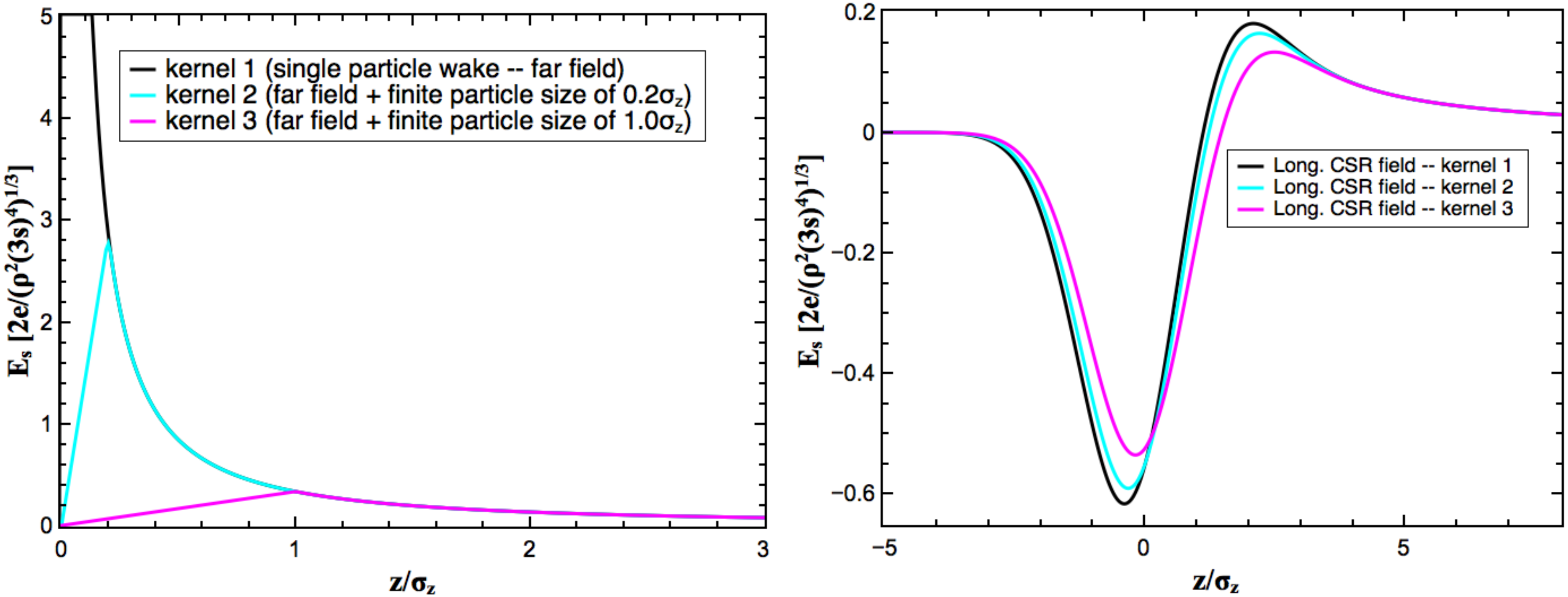
$$\left(0 = \int_{-\infty}^{s} K(s-s') ds' \ \text{ from basic principles} \right)$$

Formula first from Iogansen and Rabinovich JETP, 1960

This formula ignores all the stuff within the brackets (area goes as $\sim\gamma$) and the non-asymptotic stuff to the right of the zero crossing.

So we missing stuff that scales as γ and N^2 if it acts coherently, motivating us to investigate this further.

Single-Particle Wake Function Verifies 1-D Approximation



1-D analytic forms focus on the long range wake – divergent short range effect has little effect on overall wake for a Gaussian shape

Basic CSR Theory

We start with:

$$\frac{d}{dt}(\gamma m\dot{r}) = e\left(\vec{E} + \vec{v}\times\vec{B}\right)_r + \frac{\gamma m v_\theta^2}{r}$$

which leads to:

$$F_r = e\left(-\frac{\partial}{\partial r}\phi - \frac{\partial}{\partial t}A_r + v_\theta \frac{\partial}{\partial r}A_\theta + \frac{v_\theta}{r}A_\theta - \frac{v_\theta}{r}\frac{\partial}{\partial \theta}A_r\right)$$

We can write that as: $F_r = F_{r,vel} + F_{r,rad}$

(we separate into *vel* and *rad* components because later comparison with L-W solver)

$$F_{r,vel} = e\left(-\frac{\partial}{\partial r}\phi + v_\theta \frac{\partial}{\partial r}A_\theta\right)$$

$$F_{r,rad} = e\left(\frac{v_\theta}{r}A_\theta - \frac{d}{dt}A_r\right)$$

"usual" space-charge forces

centrifugal space-charge force

convective radiation term

Basic CSR Theory, cont.

Radial equation of motion about an equilibrium orbit becomes:

$$\gamma m\ddot{x} + \dot{\gamma} m\dot{x} = \frac{\gamma m v_\theta^2}{r} + e\frac{\partial}{\partial x}\left(-\phi + v_\theta A_\theta\right) - e\frac{d}{dt}A_r + e\frac{v_\theta A_\theta}{r} - ev_\theta B_{ext}$$

$$\boxed{r = R + x}$$

The energy deviation of a particle is:

$$\gamma_1 = \frac{e}{mc^2}\int\left(-\frac{1}{r}\frac{\partial\phi}{\partial\theta} - \dot{A}_\theta\right)ds = \frac{ev_\theta}{mc^2}\int\left(-\frac{1}{r}\frac{\partial\phi}{\partial\theta} - \frac{1}{v_\theta}\frac{\partial\phi}{\partial t} + \frac{1}{v_\theta}\frac{\partial\phi}{\partial t} - \dot{A}_\theta\right)dt$$

$$= \frac{ev_\theta}{mc^2}\int\left(-\frac{1}{v_\theta}\frac{d\phi}{dt} + \frac{1}{v_\theta}\frac{\partial\phi}{\partial t} - \dot{A}_\theta\right)dt$$

Which leads to:

$$\ddot{x} = -\frac{v_\theta^2 x}{R^2} + \frac{e}{\gamma_0 R m}\left(\left(-\beta_\theta^2\phi + v_\theta A_\theta\right) + \beta_\theta^2\int\left(\dot{\phi} - v_\theta\dot{A}_\theta\right)dt + R\frac{\partial}{\partial x}\left(-\phi + v_\theta A_\theta\right) - \frac{dA_r}{dt}\right)$$

geometrical focusing

Cancelation of CSCF and potential depression

CSR term from potential depression

"usual" space-charge forces

Convective radiation term

Writing in Terms of L-W Forces

We can collect terms and rewrite them as what is solved by the L-W fields:

$$\ddot{x} = -\frac{v_\theta^2 x}{R^2} + \frac{1}{\gamma_0 m}\left(F_{CSCF} + \frac{\beta_\theta^2}{R}\int F_{z,rad}\, ds + F_{r,vel} + \left(F_{r,rad} - e\frac{v_\theta}{r}A_\theta \right)\right)$$

$$F_{CSCF} = \frac{e}{R}\left(-\beta_\theta^2 \phi + v_\theta A_\theta \right)$$

Emittance growth estimate from a radial force of F_{net} over a bend angle α:

$$\Delta\varepsilon_n = \frac{F_{net,rms}\alpha R}{mc^2}\sigma_x$$

Emittance growth estimate from an axial force of F_z over a bend angle α:

$$\Delta\varepsilon_n = \frac{F_{z,rms}\alpha^2 R}{2mc^2}\sigma_x$$

CSR Force Representations

Can improve 1-D model (Mayes, Sagan), but here we instead look at how accurate it is for the idealized case (the Goldreich solution).

We use Ryne's code LW3D – an exact Lienard-Weichert solver with 6B source electrons, solving (up to quad precision):

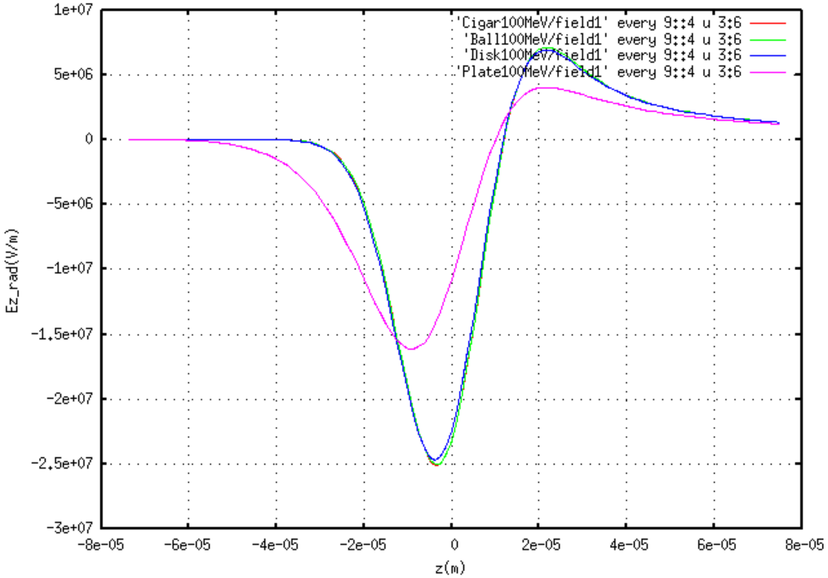
$$\vec{E} = e\left(\frac{\hat{n}-\vec{\beta}}{\gamma^2\left(1-\vec{\beta}\cdot\hat{n}\right)^3 R^2}\right)_{ret} + \frac{e}{c}\left(\frac{\hat{n}\times\left(\hat{n}-\vec{\beta}\right)\times\dot{\vec{\beta}}}{\left(1-\vec{\beta}\cdot\hat{n}\right)^3 R^2}\right)_{ret}$$

$$\vec{B} = \hat{n}\times\vec{E}$$

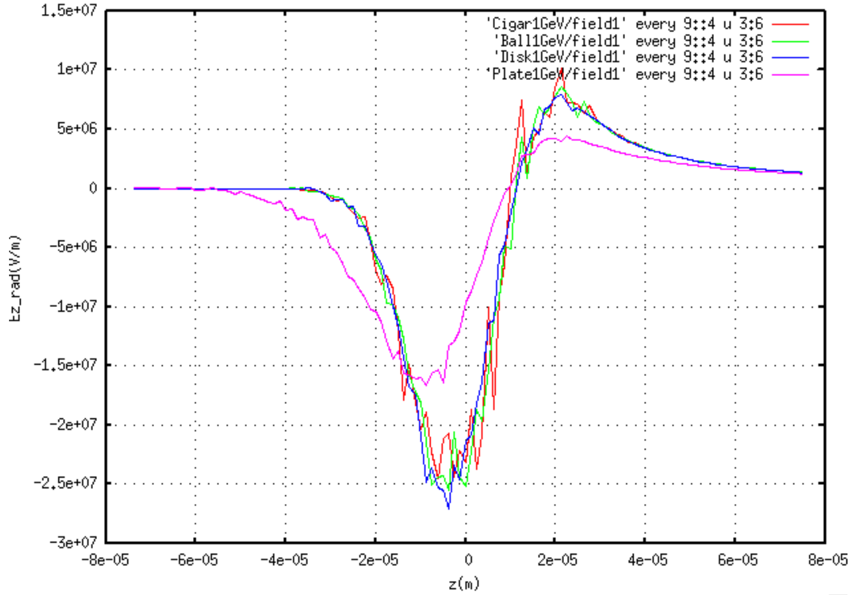
velocity field   radiation field

Needs ~ 50k processors for 1/3 hour for 1 time step (NERSC). This is not a good tool for design work, but provides a benchmark to check the physics, the 1-D model, and other effects.

LW3D Results



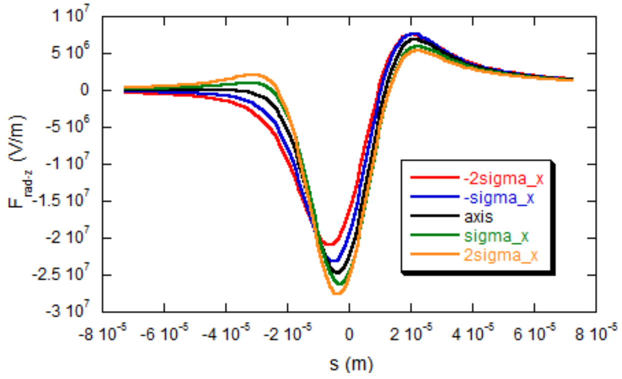
100 MeV



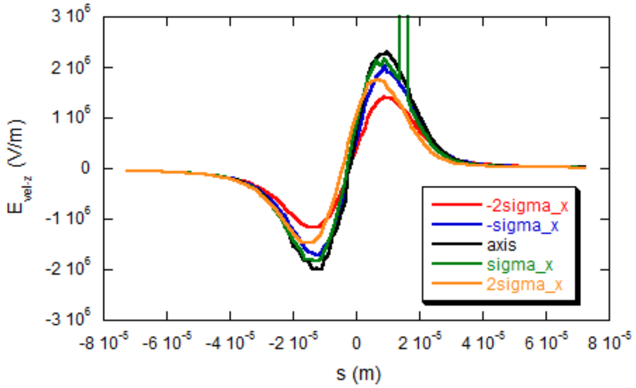
1 GeV

On axis z-directed radiative field (the CSR field) for a 10-micron long bunch (cigar: 1-micron radius, ball: 10-micron radius, disk: 100-micron radius, plate: 0.5-mm radius), all for a 1-m bend radius. The 1-D approximation is surprisingly robust (we'll get to the noise at 1 GeV a bit later).

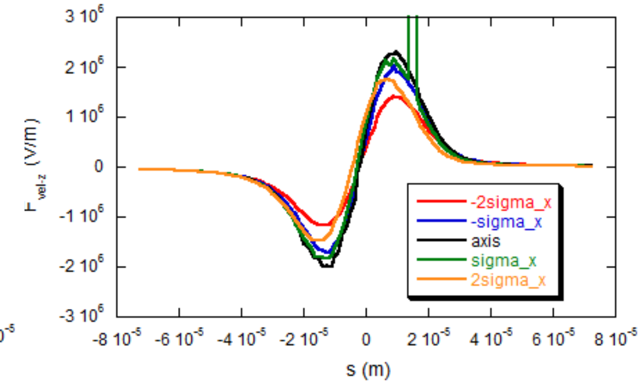
LW3D Results – 100-MeV Disk (100x10 microns)



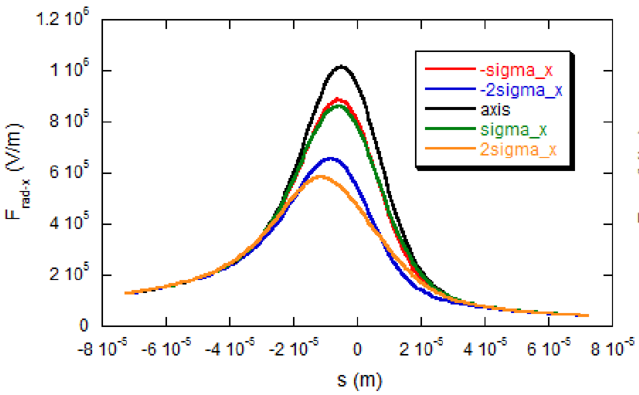
F_z-rad



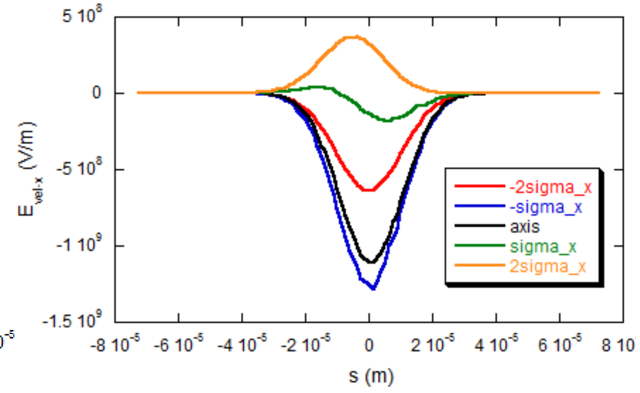
E_z-vel



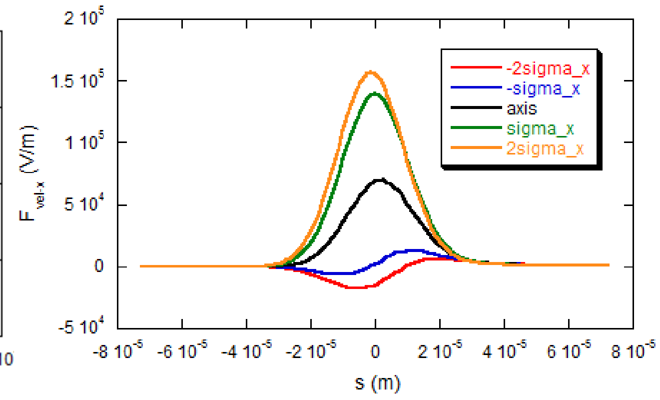
F_z-vel



F_x-rad



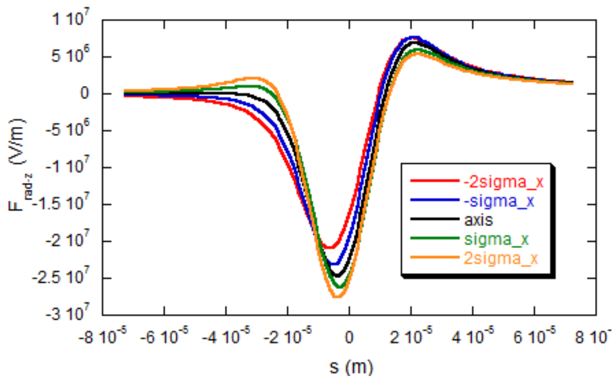
E_x-vel



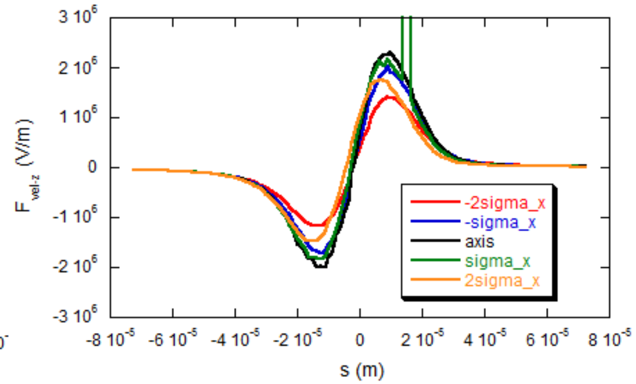
F_x-vel

Some radial spread, see expected cancelations
Plots are longitudinal cuts at different beam radii

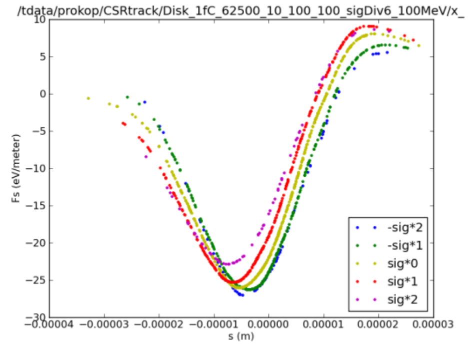
CSRTrack Has Good Agreement with LW3D

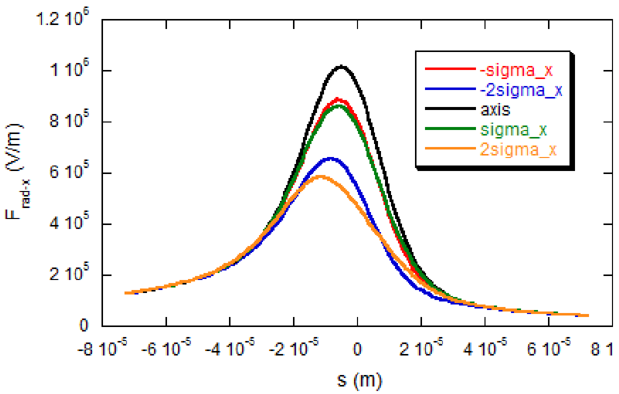


F_z-rad

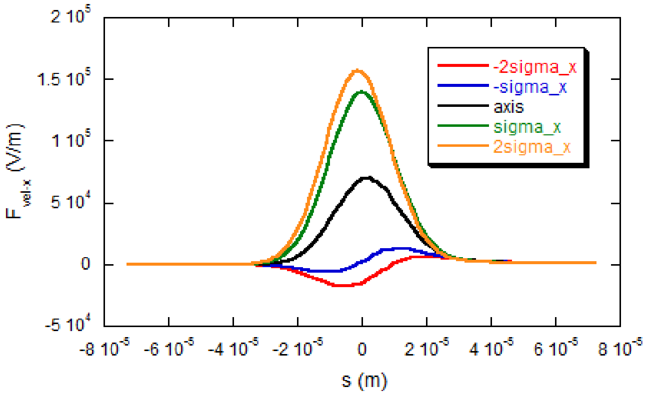


F_z-vel

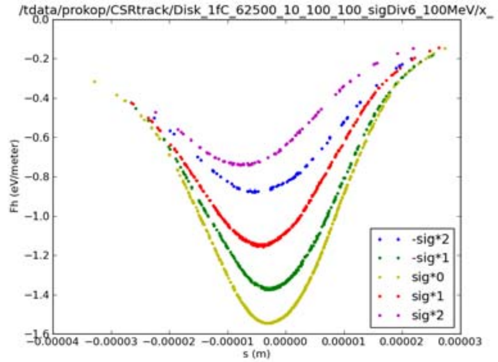




F_x-rad

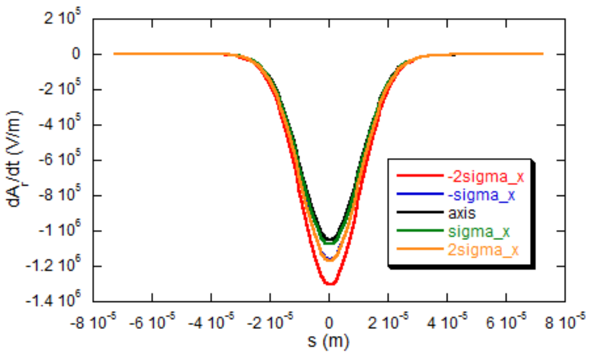


F_x-vel

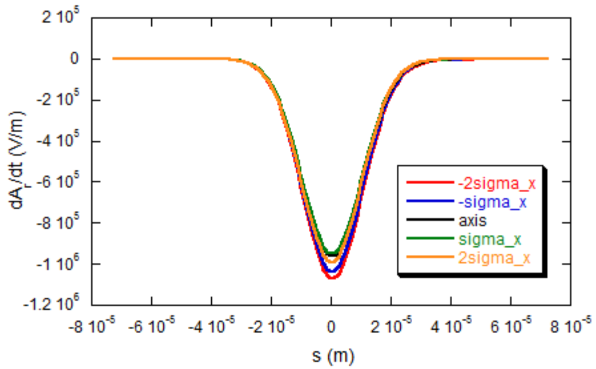


Different sign for the transverse force, but magnitude comparison is good

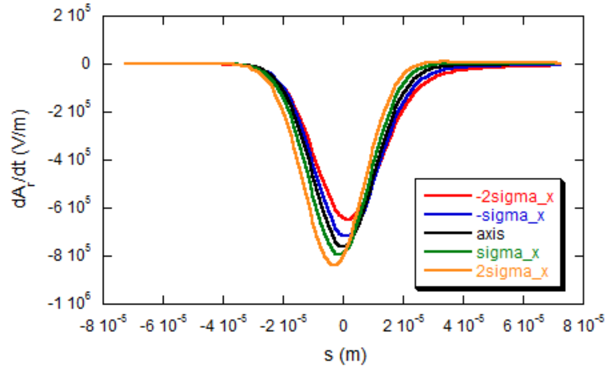
Convective Derivative is Not Insignificant



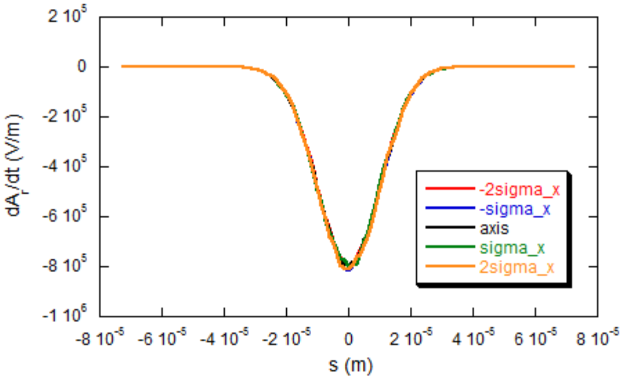
cigar



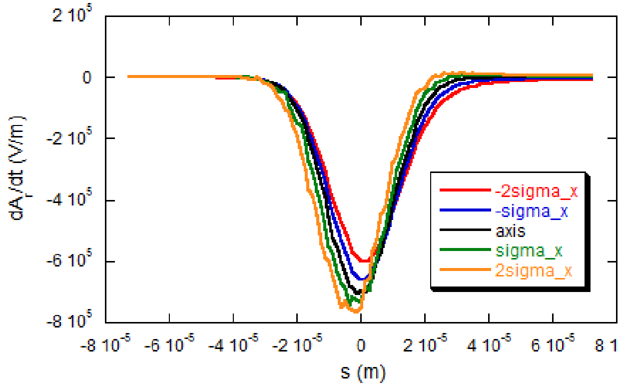
ball



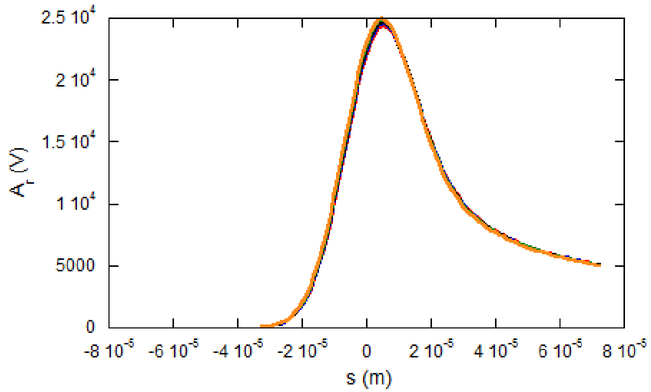
disk



cigar – 1 GeV



ball - 1 GeV

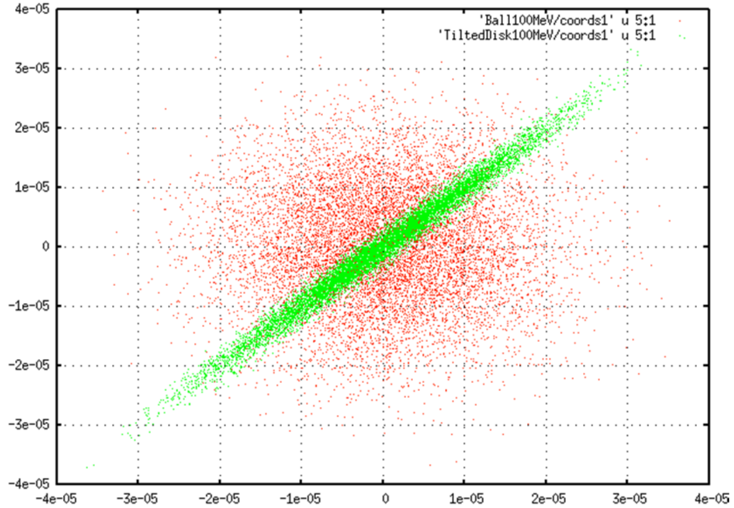


ball

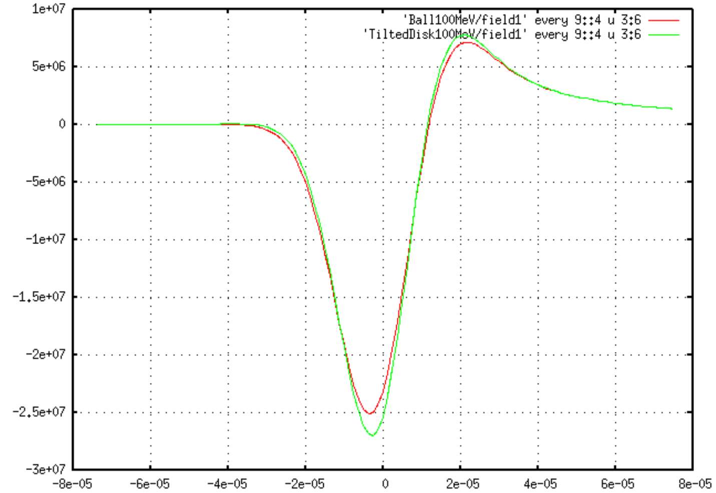
$$-\frac{dA_r}{dt} = \left(F_{r,rad} - e\frac{v_\theta}{r} A_\theta \right)$$

This net transverse field leads to a possible emittance growth comparable to that from CSR

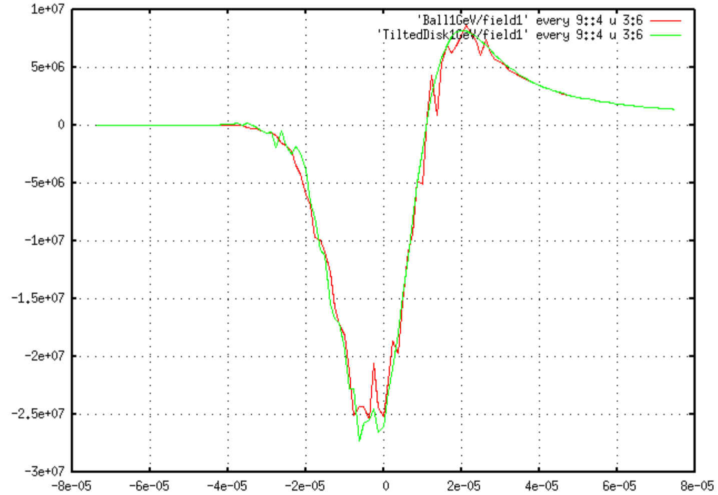
1-D Formula Works for Tilted Disk



Tilted disk has same rms transverse and longitudinal size as the 10-micron ball. The axial radiation fields are very nearly the same.



100 MeV



1 GeV

Characteristic Transverse and Longitudinal Coherence Lengths

Expect to see shot noise when the inter-electron spacing starts to approach the characteristic ISR wavelength (~ micron at 100 MeV, ~ nm at 1 GeV, inter-electron spacing ~ 10 nm)

$$\lambda_C = \frac{2\pi c}{\omega_C} = \frac{3\pi R}{\gamma^3} \qquad \omega_C = \frac{2c}{3R}\gamma^3$$

Opening angle of collective radiation is:

$$\theta_c = \frac{1}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/3} = \left(\frac{\sigma_z}{3R}\right)^{1/3} \qquad \omega = 2\pi \frac{c}{\sigma_z}$$

Maximum radius that the opening angles overlap:

$$x_c \equiv R\zeta_{ret}\theta_c \approx R\left(\frac{24\sigma_z}{R}\right)^{2/3} \frac{1}{(144\pi)^{1/3}} = 0.13R\left(\frac{24\sigma_z}{R}\right)^{2/3}$$

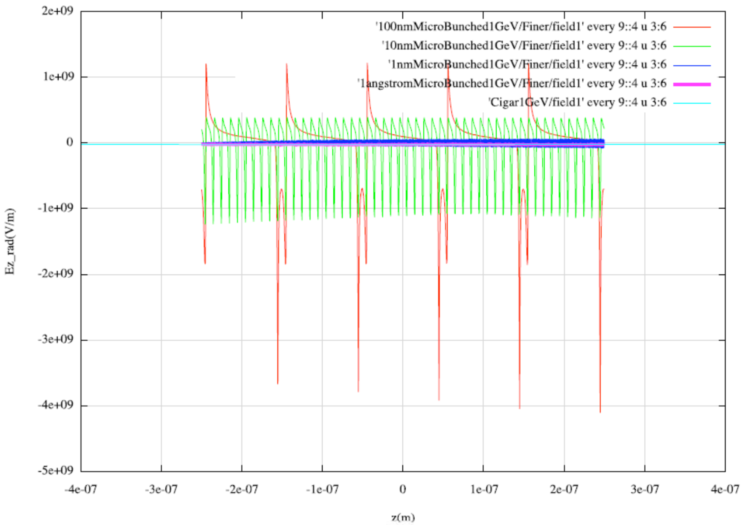
which is ~ ½ mm for a 10-micron long bunch.

For a smooth distribution, it doesn't matter much what the coherent length is, the Goldreich formula results. But there are collective effects if the distribution isn't smooth (shot noise or microbunching).

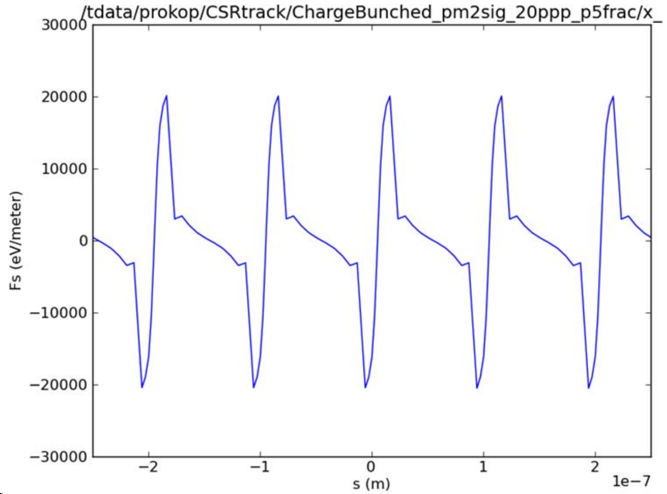
CSR Enhancement from Microbunching

Significant enhancement from a microbunched beam at 1 GeV (no effect at 100 MeV) – two orders of magnitude increase for microbunch wavelength of 100 nm. Light blue line is Gaussian CSR reference force.

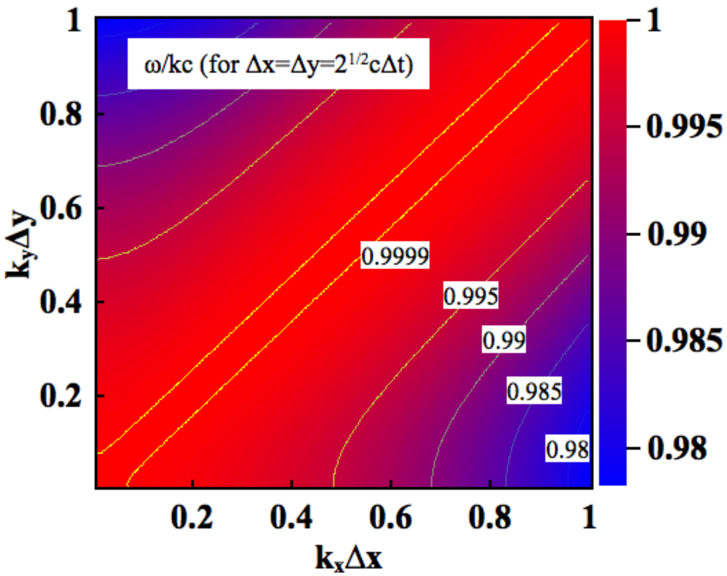
CSR is the coherent low-frequency part of the ISR spectrum, but microbunches make the higher-frequency components leading to the field singularity coherent also.



CSRTrack simulation with 100 nm structure (1 fC) (somewhat different microbunching) →



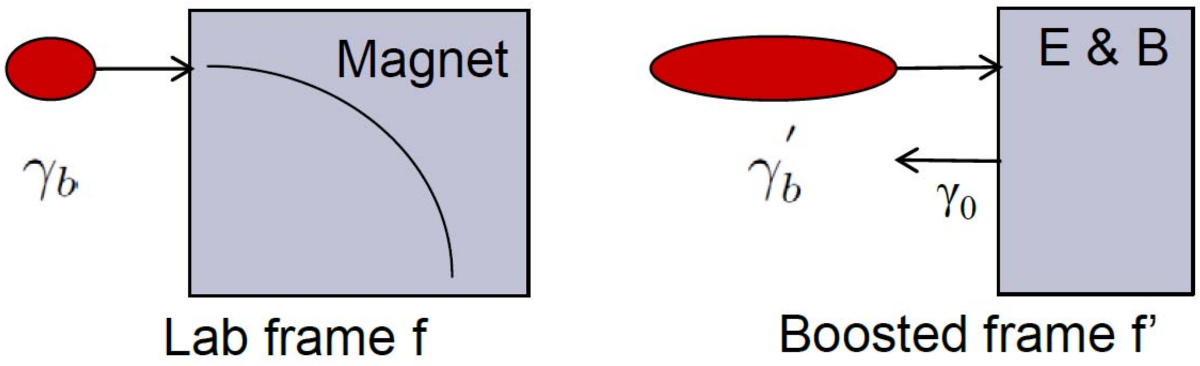
FDTD PIC Simulations of CSR



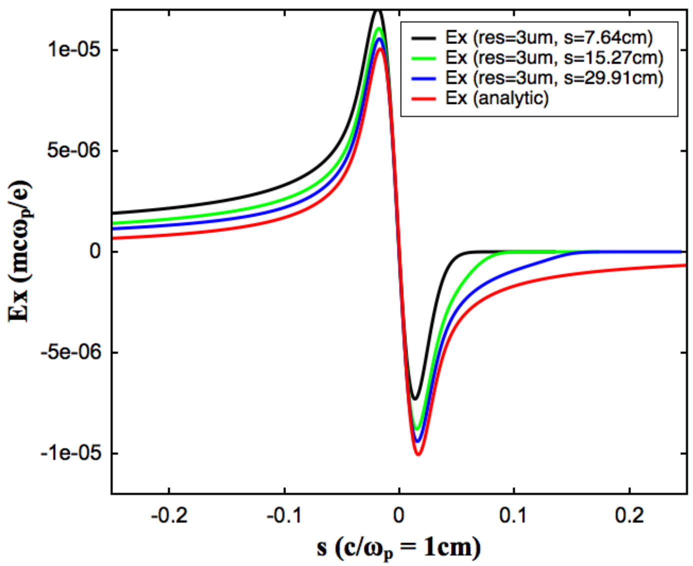
Main issue with straight FDTD PIC simulations is numerical dispersion – speed of light is < *c*, which masks real CSR effects.

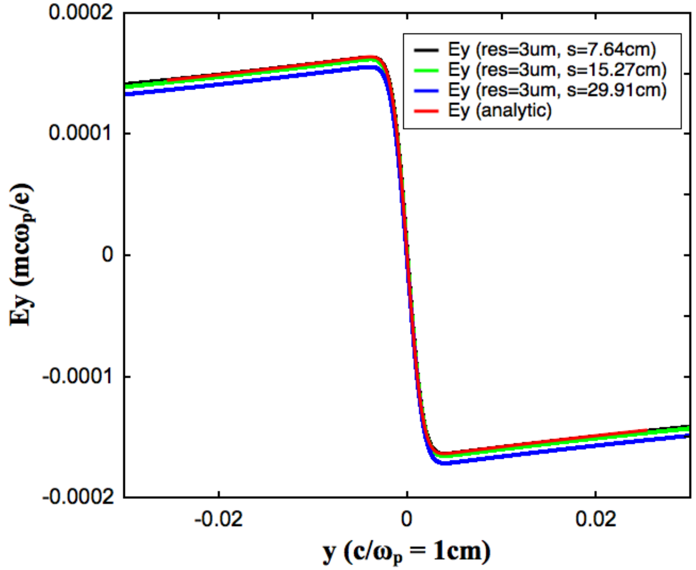
Solution (Fawley and Vay) is to go to a boosted frame:

EM wave phase velocity in 2D Yee-FDTD PIC



Boosted Frame Reproduces Straight-line SC Fields





50 MeV, γ=5 boost frame (γ'_b=10 in boost frame), bunch length increases

$$\gamma'_b = \gamma_b \gamma_0 (1 - \beta_0 \beta_b)$$

Promising approach for FDTD CSR simulations, but more work needed.

Summary

- 1-D approximation is very robust, even for pathological bunch distributions.
- Transverse force (convective derivative of A_r) can lead to an emittance effect as large as the CSR force.
- 1-D approximation loses all short-range collective (energy-dependent) effects, which may lead to significantly enhanced CSR for microbunched beams (and affect the MBI).
- CSRTrack does a very good job with CSR and transverse forces for cigar, ball, and disk shapes.
- FDTD PIC simulations have intrinsic problems (numerical dispersion), which can be mitigated with boosted frames (or techniques under development at PSI) – may lead to an intermediate level tool for CSR analysis.