Table of Contents

Definition of Emittance

The normalized root mean square (RMS) emittance in 6 dimensions is defined as

$$\epsilon_{rms} = \frac{1}{m_\mu}\sqrt{|V|}$$

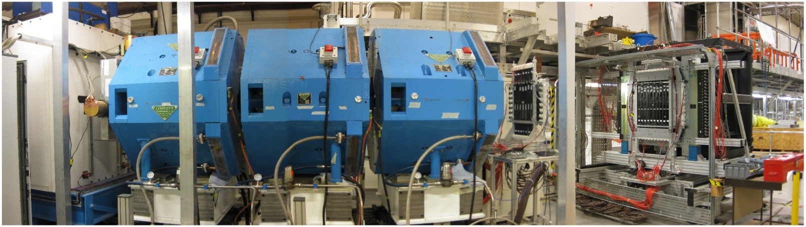
$|V|$ is the determinant of the 6×6 covariance matrix of the phase space vector

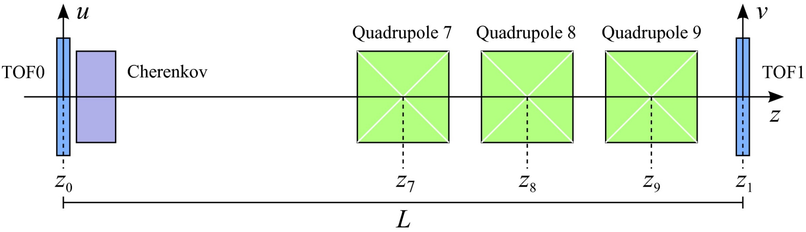
$$\vec{U} = (\vec{x}, \vec{p})$$

where $\vec{x} = (x, y, t)$ and $\vec{p} = (p_x, p_y, E)$.

- All these 6 variables will be measured in spectrometers before and after cooling cell on a particle-by-particle basis and then bunched to up to 10^6 particles for emittance calculation.
- The beam before colling channel can be measured by timing detectors. Data from TOF0 and T0F1 were used already to analyze the performance of the existing MICE muon beam line.

Measuring Emittance with Upstream TOF system



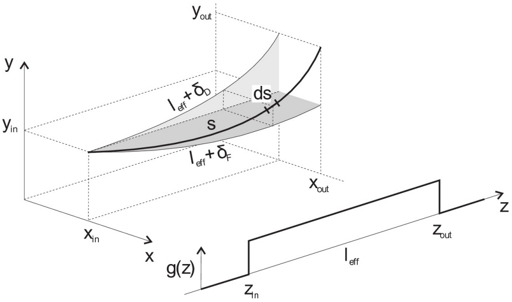


Estimation of the Momentum

The average momentum between the TOFs is given by

$$p(s,t) = \frac{m_0 s/t}{\sqrt{1 - s^2/(ct)^2}},$$

where the path length $s = L + \delta$ is reconstructed by tracking the particle's trace-space vectors $(x, dx/dz)$ and $(y, dt/dz)$ through the beam line, and integrating the path length through each section. The initial trace space vector at TOF0 can be transported to TOF1 by a transfer matrix $(x_1, x_1') = M(x_0, x_0')$ defined by quadrupole parameters. Since the TOFs provide a measurement of (x_0, x_1) and that det M = 1 for linear transformation, it is possible to find the angles $x' = dx/dz$ and $y' = dy/dz$ needed for path length calculation:



$$\begin{pmatrix} x_0' \\ x_1' \end{pmatrix} = \frac{1}{M_{12}} \begin{pmatrix} -M_{11} & 1 \\ -1 & M_{22} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \end{pmatrix}$$

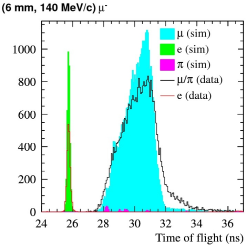
A set of beam line optics configurations have been generated and corresponding beam parameter measured by TOF system. All variables measured in data have been compared with simulation.

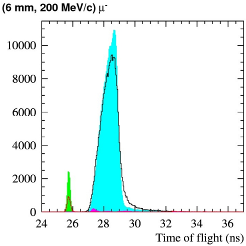
Comparison Between Data and Simulation

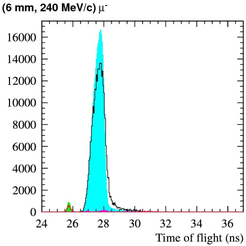
Time of Flight and Momentum

p_z = 140 MeV/c p_z = 200 MeV/c p_z = 240 MeV/c

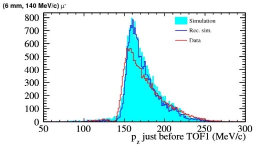
Time of flight shows π cannot be distinguished from μ, but at < 5%

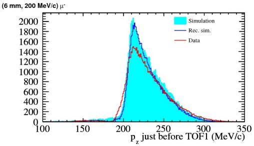


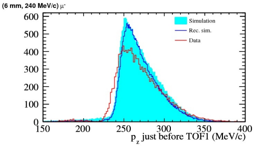




Momentum distributions also agree quite well with simulation

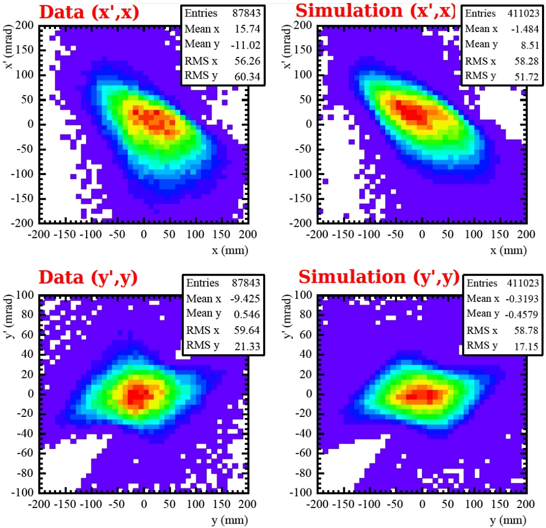






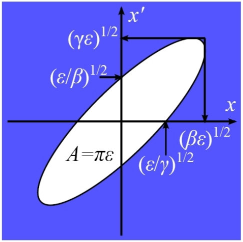
Comparison Between Data and Simulation

Transverse Trace Space



Now calculate:

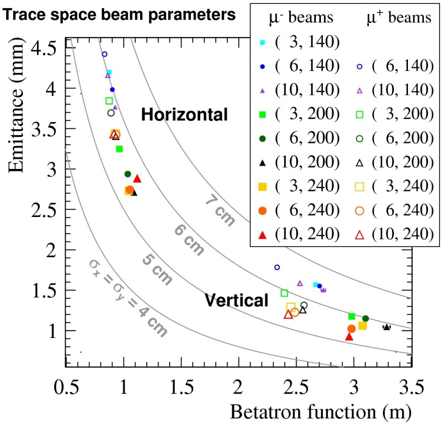
- $\epsilon_x = |\mathrm{Cov}(x, x')|$
- $\beta_x = \sigma_x^2 / \epsilon_x$



And in 4D:

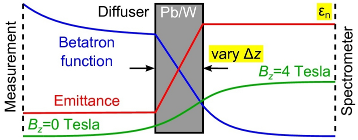
$$\epsilon_n \approx (p_z/m_\mu)\sqrt{\epsilon_x \epsilon_y}$$

Measured Optical Beam Parameters



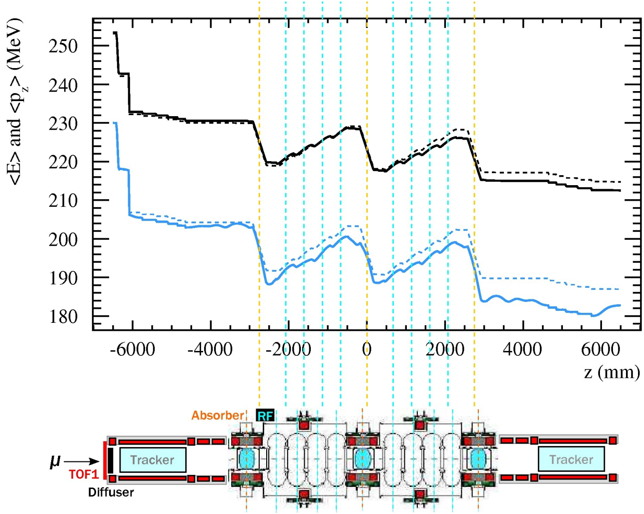
For each beam β_x, ϵ_x and β_y, ϵ_y and a contour of constant beam size $\sigma_i = \sqrt{\beta_i \epsilon_i}$ are plotted.

Beams designed to have every combination of ϵ_n = (3, 6, 10) mm upstream of the first liquid hydrogen absorber, and p_z = (140, 200, 240) MeV/c in the center of each absorber. Muon beams of both polarities were generated and key optics parameter measured at TOF1. The transverse normalized emittance is related to the measured values of ϵ_x and ϵ_y as $\epsilon_n \approx (p_z/m)\sqrt{\epsilon_x \epsilon_y}$. The emittance of the incoming beam, measured by TOF0, TOF1 is much smaller than that of the beam that will go through the cooling channel. In order to generate the desired large emittance the beam goes through a high Z diffuser of adjustable thickness situated inside the first solenoid.



Simulation of the Measured Beam in Cooling Channel

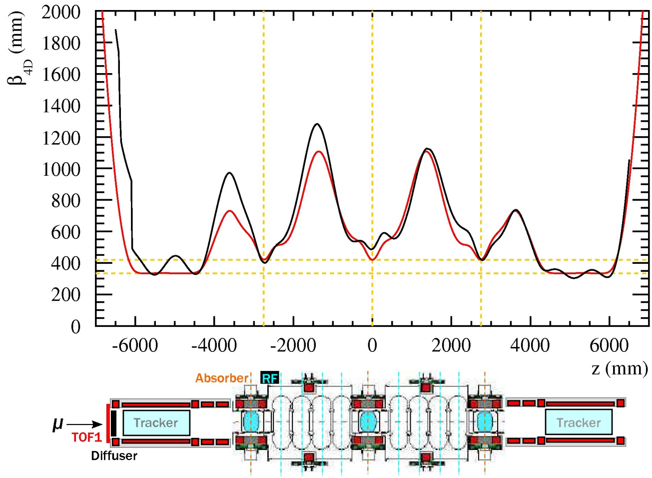
Energy and p_z



A beam, starting at TOF1, with the transverse distributions characteristic of the measured beam was simulated in full MICE cooling channel. The simulated beam was generated according to the measured covariance matrix of the four transverse phase space coordinates and therefore had the emittance and optical parameters of the real beam. *Dashed line is the reference particle traveling along the axis.*

Simulation of the Measured Beam in Cooling Channel

A Predicted Beam Envelope



The evolution of the transverse beta function along the MICE cooling channel of the real beam has been also simulated. The optics of the beam are seen to be similar to the ideal optics (red line) derived from a numerical solution to betatron equation:

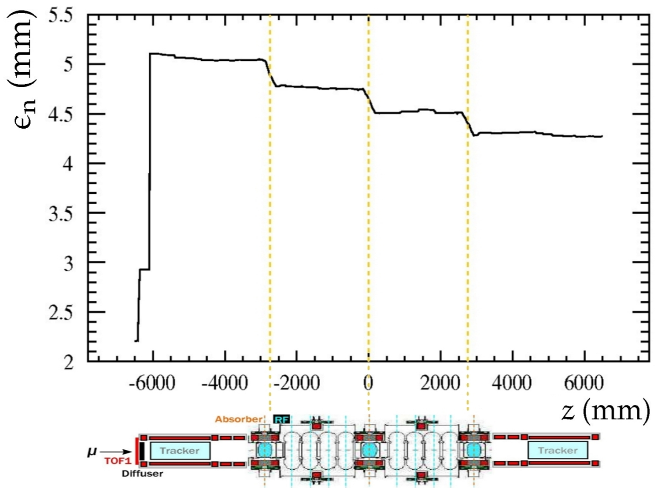
$$2\beta_\perp\beta''_\perp - (\beta'_\perp)^2 +$$

$$+4\beta_\perp^2\kappa^2 - 4 = 0$$

where κ is the focusing strength.

Simulation of the Measured Beam in Cooling Channel

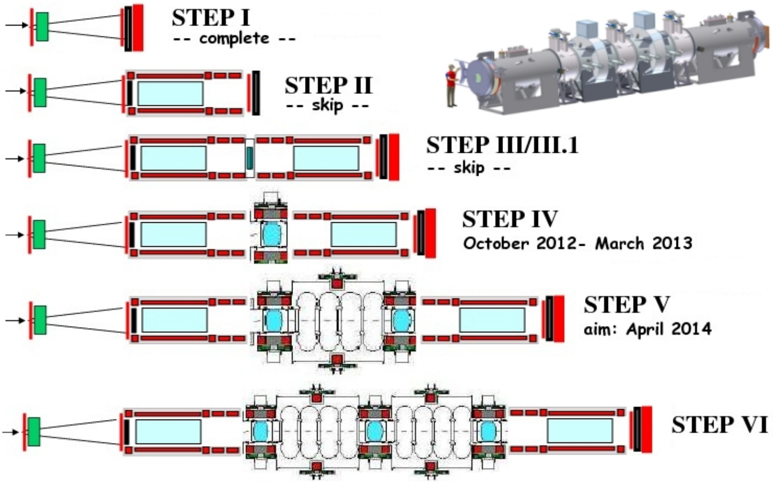
Emittance



The normalized emittance of the simulated real beam behaves as expected. The initial inflation is due to defuser. The emittance reduction in liquid hydrogen absorbers is clearly visible.

Table of Contents

MICE Schedule



Summary

- Full beam line installed and commissioned at RAL
- Particle identification detectors installed in MICE hall and used for the first emittance measurements
- Most of the components of the cooling channel produced and under commissioning
- Beams containing > 95% $\mu^{\pm}$ generated for ϵ_n = (3,6,10) mm and p_z = (140,200,240) MeV/c
- Distributions of the trace space vectors of individual muons have been reconstructed for the various beams, and promising agreement is observed with Geant4 simulations
- The evolution through MICE of the optical parameters of a measured baseline beam has been simulated and the beams are relatively well matched
- MICE is on the way towards demonstrating ionization cooling

Thank you for your attention!